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Study on Natural Frequency of Frame Structures

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ABSTRACT

Moment resisting frame (MRF) structures are gaining popularity for their high lateral stiffness. This study investigates the parameters which affecting the natural frequency of moment resisting frame structures. Steel and concrete MRF structures were studied theoretically, analyzed numerically to obtain their mode shapes and frequency of vibration for each mode. From the theoretical and analytical results, a model equation for approximation of natural frequency of these types of MRF structures is proposed. The proposed model expressed the relationship of natural frequency of MRF structure with its total mass, lateral dimension in the direction of vibration and total height. The proposed equation will be helpful and easy to calculate the fundamental frequency for study on dynamic behavior of structures. Comparison between the current guidelines and proposed model is also discussed. The proposed model is satisfying the general concept of free vibrational response, and can be applied for analyzing small and full scale structures.

1. Introduction

The analysis of dynamic forces and study on the response of structures during and after any applied excitation is very challengeable. To understand the dynamic behavior of structures, investigations on small-scale prototype models are very important and useful to draw general

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conclusions. Dynamic analysis of any structure involves with the equation of motion of the structure, which is needed to analyze in order to find the modes of vibration with the corresponding natural frequencies [1]. It is because, the determination of the lateral loads caused by seismic activity depends on the fundamental period of the structure, which is needed to be estimated theoretically or experimentally [2]. In addition, fundamental frequency is an important tool for dynamic design and does not involve with heavy dynamic testing [3]. Free vibration analysis can be applied to determine the natural frequency and modes of vibration of structure, which can be done experimentally or numerically. Additionally to record the free vibrational response, numerical modal analysis is simplest method which is generally being applied [4].

Besides reinforced cement concrete (RCC), steel structures gaining popularity for their high ductile nature. Additionally, moment resisting frame (MRF) structures having high lateral stiffness and appreciable resistance to seismic activity. Different types of MRF structures were used by the researchers to find more reliable structures under seismic load. Structures deflected more due to reach in resonance condition under short time while vibrating. To avoid the resonance problem in high rise building, the fundamental Eigen frequency should be maximized [5]. To decrease the natural period of irregular planned building additional steel frame was introduced parallel by Vail (2017) [6]. Researchers using intermediate and special MRF structures to withstands these problems also [7]. Extensive researches found on fundamental period of reinforced cement concrete moment resisting frame structures [2,8–11]. Most of them included height as a function of fundamental period of structures. But, other properties of structures are also showing influences on fundamental period. Meanwhile, there are few researches on fundamental period of steel MRF structures. Therefore, this study is an attempt to count the considerable properties of MRF structures that have direct influence on fundamental frequency of structure. Based on the previous research, natural period of MRF structure is not so dependent on the number of spans [8]. Therefore, the effect of lateral dimension in the direction of external vibration, mass and height of steel MRF structures on natural frequency were analyzed in this study. Because, the lateral dimension possesses significant impact on the stiffness of MRF.

Besides experimental program, numerical programming and analysis using universal software are also effective to come in conclusion on structural behavior. Researchers using different computer programming and software to simulation of the structures with loading varieties. For example, Michał (2007) [12] has done free vibrations analysis based on the boundary element method, where non-singular approach was used for analyzing a thin plate. Nilesh and Desai (2012) [13] studied about the variation in natural frequency with varying numbers of floors of RCC building, where STAAD-pro software was being used in the analysis. In the study of Pavol et al. (2013) [14], modal analysis of titan cantilever beam was conducted using ANSYS and SolidWorks software. Therefore, dynamic analysis of structures using different types of universal software with various parameters is helpful and contributory to the field of dynamic study.

This study is based on modal analysis of MRF structures using ANSYS software to study the natural frequency and deflection pattern with their controlling parameters. A simplified model equation for prediction of natural frequency of MRF structures is proposed, with considering the structure height, lateral dimension and mass of the MRF structure simultaneously. This model will be helpful for prediction of natural frequency easily and can be applied for different types of materials.

2. Theoretical background

Due to the earthquake motion, building vibrates. For free vibration of the building structure, a complete back and forth movement requires total natural period of time. According to the thumb rule provided in FEMA guideline [15], a full scale building's natural period is equals to □th of the number of stories [15]. Thus the building's height is the most crucial factor. Additionally, some other factors related to the mass and stiffness of structures, such as, structural system and configuration as well as materials also affect the period [16]. The first mode of free vibration is critical and resonance risk is high. However, the other modes of vibration could be critical for high-rise structure, which are assumed less critical than the natural period for low-rise building [17]. Thus the fundamental frequencies of structures need to be analyzed. Natural period that calculated using the current guidelines are proven as underestimated, thus give lower values due to the assumption of rigid joints in frame structures [1]. Though the fundamental period is a parameter for estimating the base shear for a structure, thus it should be sound accurate in order to prevent unsafe design.

There are several other simplified formulas devolved and standardized with code. As per IS 1893:2002 [18] guidelines, the fundamental period of vibration (T_a) of a MRF structure can estimate by using the empirical expression in Equations (1) and (2):

For RC frame building,

$$T_a = 0.075\ h^{0.75}\ \text{sec} \tag{1}$$

And for steel frame building,

$$T_a = 0.085\ h^{0.75}\ \text{sec} \tag{2}$$

Where, h = Height of building in meters (excluding basement storey if basement walls are connected with the ground floor deck or fitted between the building columns, otherwise includes). The MRF is considered without brick infill.

Similarly, according to BNBC 2014 [19], T_a of a MRF building can be estimated by the empirical relation in Equations (3) and (4).

For concrete frame,

$$T_a = 0.0466\ h^{0.90}\ \text{sec} \tag{3}$$

And for steel frame,

$$T_a = 0.0724\ h^{0.80}\ \text{sec} \tag{4}$$

Where, h= Height of structure in meters above the base

The way of expression of the fundamental period of the MRF frame given by IS 1893:2002 [18] and BNBC 2014 [19] are same, but the coefficients are different which is idealized based on current design guidelines and other structural factors in local areas. However, in both cases, lateral dimensions and mass of building are not included though the stiffness and mass of any structure are the main factors controlling the natural frequencies.

Meanwhile, theoretical analysis of any MRF can be done following the typical vibration equation for single degree freedom system. If the base connection is modeled as fixed shown in Figure 1 (like the column-beam moment connection), the stiffness of the frame is defined by Equation (5) [20,21]:

$$k = \text{Number of column} \times 12EI/h_i^3 \quad (5)$$

Where, I=Moment of inertia of the column section and h_i = height of each storey

For a multistoried moment resisting frame shown in Figure 1, the total mass and total stiffness of building is used to find the natural frequency.

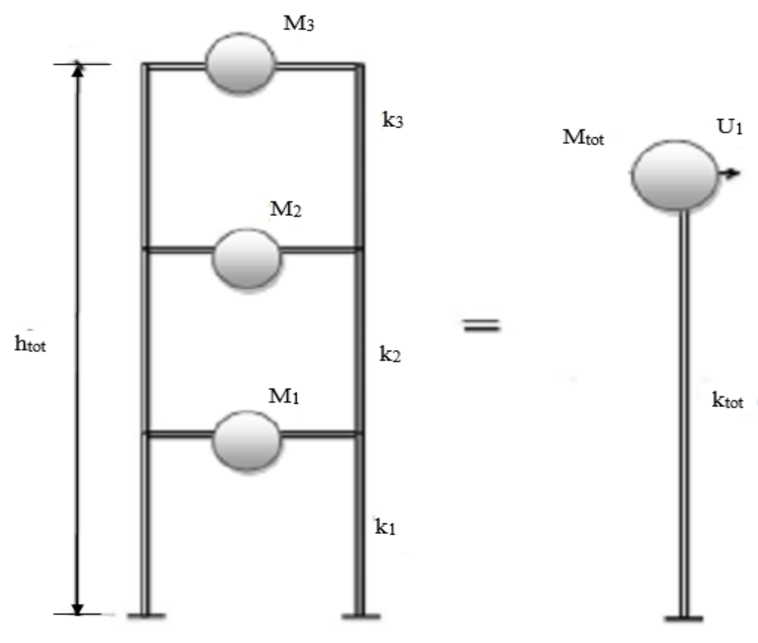

Fig. 1. Equivalent Single Degree of Freedom Frame Structure.

To find the total mass (M_{tot}) simply sum the mass of each floor (M_1, M_2, M_3). The total stiffness (k_{tot}) is slightly more difficult to calculate. The Equation (6) is used for combining the stiffness of adjacent floors is as follows:

$$k_{tot} = \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)^{-1} \quad (6)$$

Where: k_1, k_2, and k_3 are the stiffness of the first, second, and third floor, respectively. These are calculated using the equations mentioned above. Using the total mass and stiffness in the Equation (7) below, fundamental frequency (*f*) of MRF structures can calculate.

$$f = \frac{1}{2\pi}\sqrt{\frac{K_{tot}}{M_{tot}}}\ \mathrm{Hz} \tag{7}$$

3. Structural analysis

3.1. Selection of structural model

Small-scale models are useful, for studying basic principles of dynamics and structural concepts. Researchers used different types of materials for constructing models for dynamic analysis, such as wooden model [22–24], steel model [25,26] and RCC model [2,27,28] for their research. In modern civil engineering, framed structures consisting beam and column elements with high lateral stiffness, low self-weight and high ductility are preferable. Therefore, MRF structures were selected as model in this study. In this study fifteen models were studied theoretically and analyzed using ANSYS (ANSYS 11) software. The beam and column of the MRF models were modeled (Figure 2) with mild steel wire and concrete materials separately and with variable dimensions. The steel wire beam and column were circular shaped and the concrete beam and column were square in shape. Dimensions and material properties of studied models and their theoretical natural frequency obtained from Equations (5), (6) and (7) are listed in Table 1.

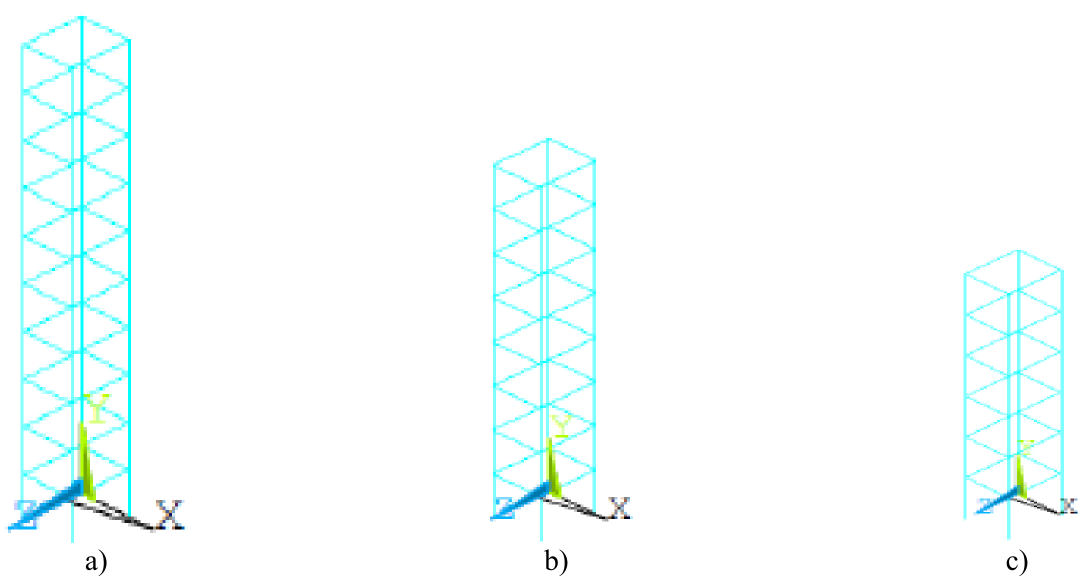

Fig. 2. a) Model 1; b) Model 6; c) Model 11.

3.2. Modelling in ANSYS software

Element BEAM4 with circular solid section CSOLID of mild steel wire was selected for the beam and column of model 1, 2, 3, 6, 7, 8, 11, 12 and 13; and for model 4, 5, 9, 10, 14 and 15 rectangular section RECT of plain concrete was selected with dimensions listed in Table 1. The models 1, 2, 3, 4 and 5 configured using 44 key points and 80 elements. Models 6, 7, 8, 9 and 10 consists of 36 key points and 64 elements and models 11, 12, 13, 14 and 15 consists of 28 key points and 48 elements.

Table 1.
Natural Frequency of Model Structures.

Model name	Column Size/ Diameter (mm)	Height of Each Floor (mm)	No of Storey	Lateral Dimension (m)	Mass of Model (kg)	Theoretical Natural Frequency of Model (Hz)	1st Mode Frequency from Modal Analysis (Hz)
Model-1	2	127	10	0.1524	0.262	5.94	5.18
Model-2	2.5	127	10	0.1524	0.401	7.44	6.45
Model-3	4.95	127	10	0.1524	1.605	14.74	12.72
Model-4	17×17	127	10	0.1524	7.01	33.73	25.49
Model-5	408×408	3048	10	3.656	101290.4	1.41	1.06
Model-6	2	127	8	0.1524	0.209	7.45	6.53
Model-7	2.5	127	8	0.1524	0.325	9.90	8.122
Model-8	4.95	127	8	0.1524	1.292	18.39	16.07
Model-9	17×17	127	8	0.1524	5.8	42.58	33.91
Model-10	408×408	3048	8	3.656	81163.6	1.76	1.41
Model-11	2	127	6	0.1524	0.139	9.91	8.80
Model-12	2.5	127	6	0.1524	0.2166	12.33	10.96
Model-13	4.95	127	6	0.1524	0.905	24.63	21.73
Model-14	17×17	127	6	0.1524	4.363	56.71	47.45
Model-15	408×408	3048	6	3.656	60741.8	2.34	1.97

The bottom end of the columns were considered as fixed support to maintaining the similarity of actual building condition. The studied models were considered as made of mild steel wire and plain concrete elements; both were assumed as isotropic and elastic material. The 1^{st} mode natural frequency of the MRF models are listed in Table 1 after completing the modal analysis using ANSYS. Calculation of fundamental frequency of moment resisting framed structures using theoretical formula and any universal software is tedious and time consuming, consisting of several calculations. To find the approximate natural frequency of these types structures an easy and empirical formula will be helpful.

4. Results and discussions

4.1. Factors affecting natural frequency

It is found that, natural frequency is an inverse function of building height. Structures constructed with same elements (beam and column) with different height showed a great variation in natural frequency. Researches showed that taller building will undergo in resonance condition with lower frequency of vibration of earthquake. Therefore, shorter building will affected by high frequency vibration of earthquake within short time. This concept is in well agreement with this study. Dimension of columns and beams are also controlling factor of natural frequency, because it relates to the stiffness and total mass of structure. Increased in beam or column diameter causes increased in mass and increased in stiffness of total structures (Figure 3). Finally the effect of mass and stiffness introduced in natural frequency of structures according to the Equation (7). A relationship can develop using the data of modal analysis and theoretical study to find approximate natural frequency of moment resisting frame structures empirically.

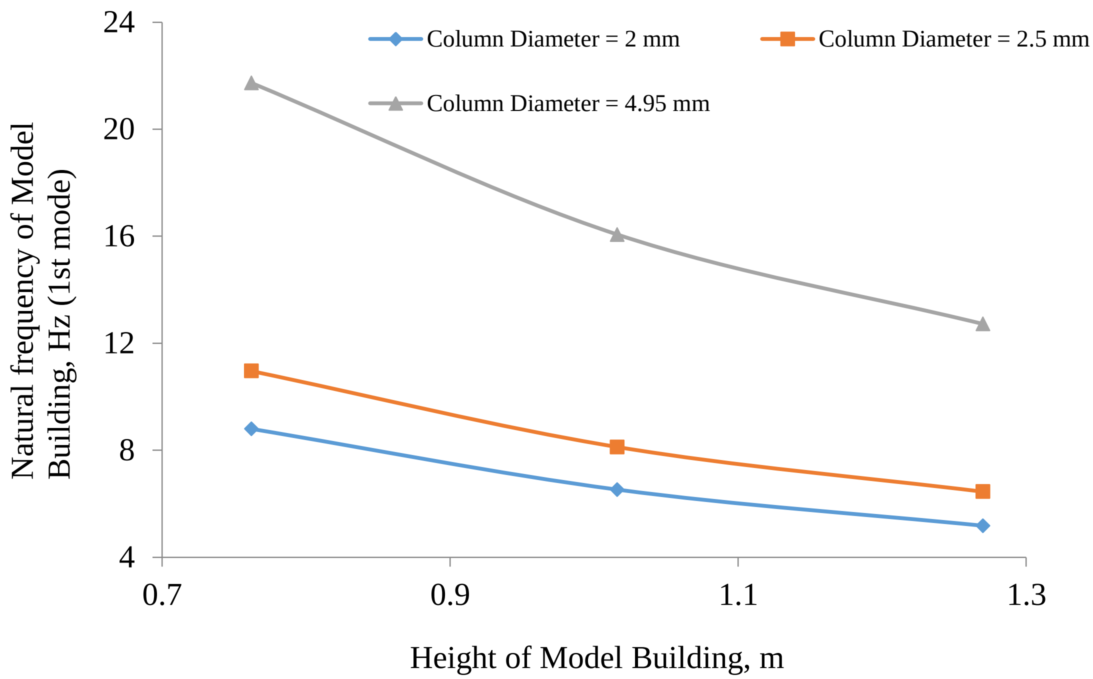

Fig. 3. Variation of Natural Frequency with Building Height.

4.2. Mode shapes of models

Mode shapes are the very important concepts to analyze the deflection pattern of structures due to dynamic excitation. In this analysis, it is confirmed that, mass and stiffness of structures having a great effect on deflection shapes. Being identical in configuration, all the model's first mode is 1st horizontal sway mode and second mode is 2nd horizontal sway mode and the 3rd one is 1st torsional mode of vibration, but their displacements are different. Model 4 is configured as 1:24 scale of model 5 in dimension. 1st three mode shapes (Figures 4 and 5) of these two models are same in pattern, but deflection is found 116:1. Because stiffness of model 5 is 24 times of model 4. Therefore stiffness controls the deflection pattern of structures under dynamic loading. It was observed that the maximum deflection found at the points on peak of the model, as example at the highest key points 41 to 44 for model 1, 2, 3, 4 and 5. That is the adverse effects of vibration increases with its height.

MODEL 4	First Mode Shape	Second Mode Shape	Third Mode Shape
	Frequency =25.49Hz	Frequency =25.81Hz	Frequency=41.435Hz
	D_{max} = 199 mm	D_{max} = 193 mm	D_{max} = 193.6 mm

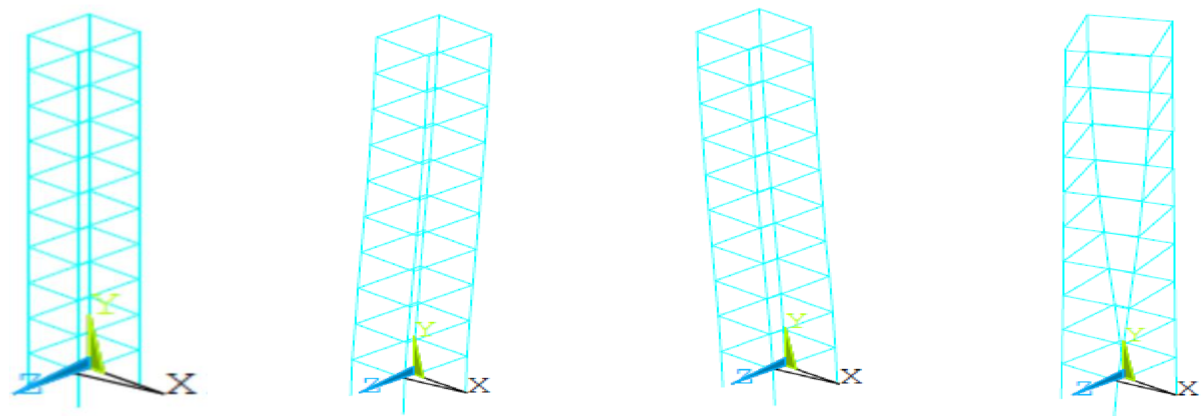

Fig. 4. Mode Shapes of Model 4.

MODEL 5	First Mode Shape	Second Mode Shape	Third Mode Shape
	Frequency = 1.059Hz	Frequency=1.072Hz	Frequency = 1.72 Hz
	D_{max} = 1.7 mm	D_{max} = 1.63 mm	D_{max} = 1.65 mm

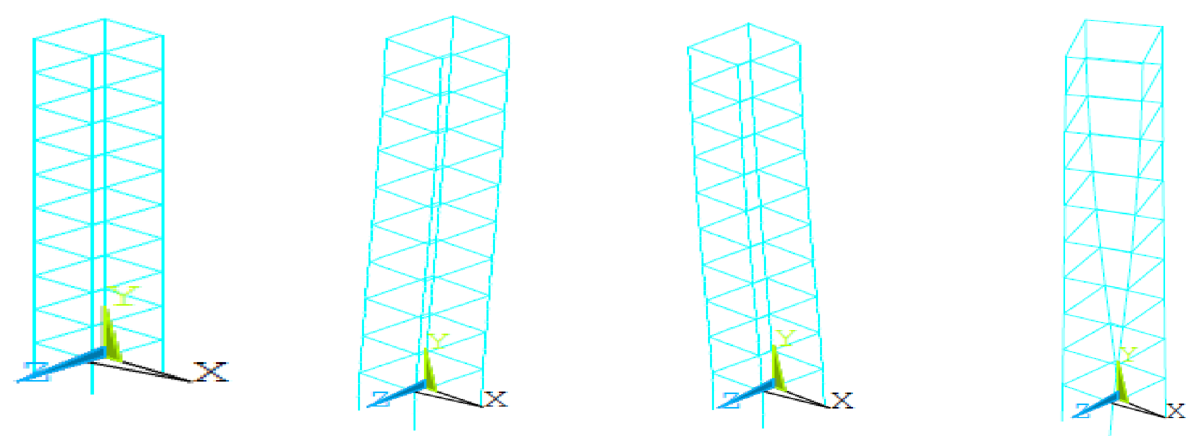

Fig. 5. Mode Shapes of Model 5.

4.3. Natural frequency of moment resisting framed structure

To determine approximate natural frequency of moment resisting framed structures a general relationship between total mass, total height of structure and lateral dimension of frame along the direction of vibration with natural frequency could be made in accordance with the results found from modal analysis of 15 structures in ANSYS software and theoretical calculation. This relationship is stands for an approximation of natural frequency of beam-column connected moment resisting frame structures. The relationship can be expressed as following Equation (8):

$$fa = 2.7 \frac{M^{0.5}}{h^{1.6}D^{0.9}} \tag{8}$$

Where, f_a is approximate natural frequency (Hz) of MRF building, M is the total mass (kg) of building, D is length (m) of frame in the direction of vibration and h is the total height (m) of building. This equation is an approximation, which is stand for typical set of study and actual natural frequency depends on building's other practical parameters also. Therefore the model should be justified. Though it was proven that the mass shows an inverse relation with natural frequency, but when incorporating the effect of mass, height and lateral length simultaneously, a different relationship is being aroused. This happened due to the increasing size of the member the stiffness and mass increases simultaneously. And may be the stiffness increment is quite large enough to overcome the reduction in frequency due to mass increment. Thus, this question need to be justified experimentally.

4.4. Justification and limitations of model

Natural frequency of structure depends upon stiffness, mass, and configuration of structural components. The proposed model stands as empirical model, which is expecting to be justified. The proposed model for approximate natural frequency of MRF structures can be justified using previously studied data. The Table 2 is stand for justification of this proposed model, which may be helpful to the field of dynamic study.

Table 2 shows that as per BNBC 2014 and IS 1893:2002 the approximate natural period of vibration is only a function of height of a building regardless the material of building. It is known that natural frequency (1/Ta) is also a function of the dimension of column (stiffness, mass) and lateral dimension of structures, not only the height of the building. Model 1, 2 and 3 were of the same height but with different sizes in column and beam. So mass and stiffness of these models were different. As per BNBC 2014 and IS 1893:2002 formulas, approximate natural frequency of model 1, 2 and 3 are same. But for being different in dimensions, the natural frequency of these three models were different. Natural frequency of all model structures calculated using guideline from BNBC 2014, IS 1893:2002, theoretical study, modal analysis and using this proposed formula were shown in a graphical representation in Figure 6. Using the proposed model the calculated natural frequencies are nearly the theoretical and numerical values of natural frequency of structures, where BNBC 2014 and IS 1893:2002 code gives different values of frequency for most of the models. Again, the calculated natural frequencies according to BNBC 2014 and IS 1893:2002 are much higher than the analytical and theoretical values for small mass frames. Therefore, it may be a helpful model besides the standard codes for approximation of natural frequency.

Table 2.
Natural Frequency of Models obtained from different Formula from Previous Study.

Source	Modal Analysis (ANSYS software)	BNBC 2014	IS 1893:2002	Proposed Formula
Natural frequency of Model 1 (Hz)	5.18	11.41	9.83	5.12
Natural frequency of Model 2 (Hz)	6.45	11.41	9.83	6.34
Natural frequency of Model 3 (Hz)	12.72	11.41	9.83	12.69
Natural frequency of Model 5 (Hz)	1.059	0.99	1.02	1.12

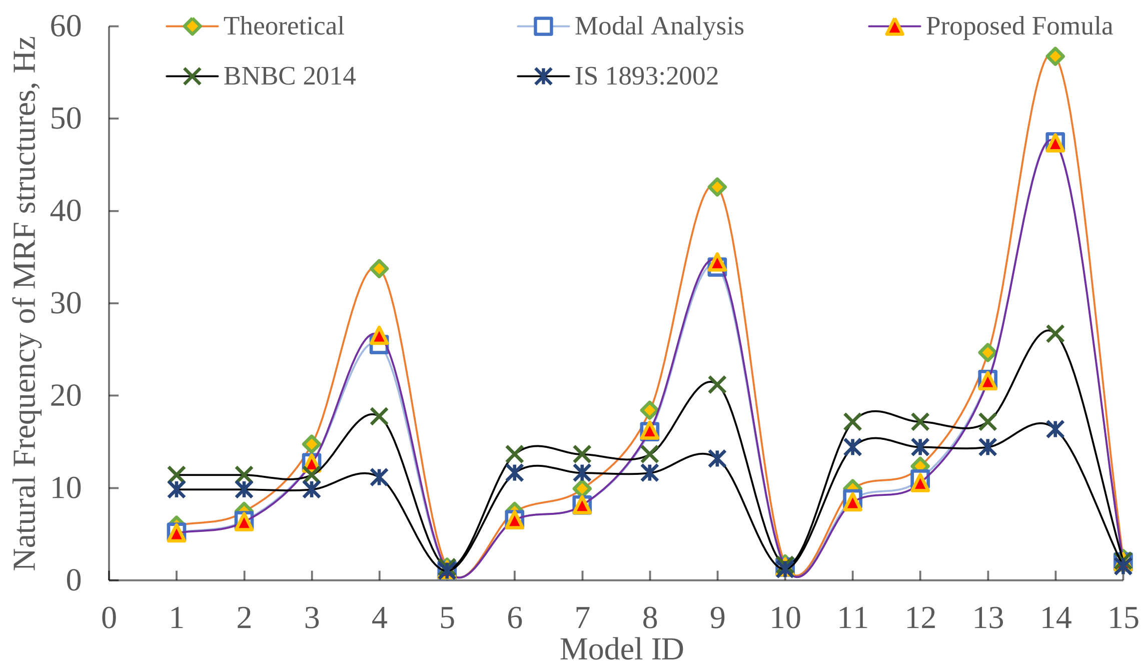

Fig. 5. Natural Frequency of MRF Structures using different Formula.

However, the proposed formula is derived using the results obtained from numerical analysis using ANSYS, and validated through the natural frequencies calculated for equivalent single degree of freedom frame structure for model frame. There were some approximation and conditions which need to be justified using experimental investigations. As this study is fully based on theoretical and numerical based, the limitations cannot be ignored. The conditions and limitations are listed as follows:

- This proposed model is based on equivalent single degree of freedom frame structure without infills.
- The model frames were symmetric MRF system. Therefore, the lateral dimensions were same for full length of MRF.
- Height of the each floor were same, but the model can be applied for dissimilar height also, and the final equation only requires total height of structure.

This model is a primary attempt to increase the parameters in the thumb equation of natural frequencies. To increase the applicability of this model and improve the accuracy of results more analysis are required. Some of the recommendations are follows:

- More model frames with variety in mass and configuration are need to be analyzed and incorporated in the model.
- The details boundary conditions and material properties need to be addressed.
- Experimental justifications need to be done to be more accurate in results and widening the applications. Question arisen from the dependency of natural frequency on mass and lateral dimension of frame simultaneously need to be resolved through experimental results.

However, in recent, several researches have been undertaking on fundamental frequency analysis of different types of structures, where different boundary conditions, joint conditions, and relating parameters are being included. The readers are directed to recent researches [29–31]. The flexure and shear behavior of structures are also being incorporated with the modelling of natural period. Thus a more accurate approximation can be possible. However, the specific models were being developed with specific conditions and their justification and simplicity should be justified before adoption for any structures.

5. Conclusion

A typical conclusions can be drawn from this study and can be applied for full scale structures. Because, the model will react similarly to the actual building if the model's natural frequency matches with the building's natural frequency. However, the conclusions of this study was in agreement with the typical behavior and free vibration frequency of moment resisting frame (MRF) structures. The very first concept is the fundamental frequency of structure decreases with decreasing stiffness and increasing height of structures. Natural frequency of building is related to the lateral dimension of structure as like the building height. Additionally the stiffness and mass of the structure is dependent on the dimension of the column elements. Thus any change in column dimension abruptly alters the dynamic behavior.

The proposed model for approximation of natural frequency of MRF structures in terms of building height, total mass and lateral dimension is quite satisfactory with the numerically and theoretically analyzed results and will be useful for further dynamic study. Future investigations on this proposed model can be effective to add more and obtain useful solutions based on wide variety of conditions and applications.

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Conflict of Interest

The authors declare that they have no conflict of interest.

References

[1] Al-Aasam HS, Mandal P. Simplified Procedure to Calculate by Hand the Natural Periods of Semirigid Steel Frames. J Struct Eng 2013;139:1082–7. doi:10.1061/(ASCE)ST.1943-541X.0000695.

[2] El-saad MNA, Salama MI. Estimation of period of vibration for concrete shear wall buildings. HBRC J 2017;13:286–90. doi:10.1016/j.hbrcj.2015.08.001.

[3] Gaile L, Tirans N, Velicko J. Evaluation of Highrise Building Model Using Fundamental. 4th Int. Conf. Civ. Eng. 2013, Proc. Part I, Struct. Eng., 2013, p. 15–20.

[4] Rezaee M, Yam GF, Fathi R. Development of "modal analysis free vibration response only" method for randomly excited systems. Acta Mech 2015;226:4031–42. doi:10.1007/s00707-015-1467-3.

[5] Alavi A, Rahgozar R, Torkzadeh P, Hajabasi MA. Optimal design of high-rise buildings with respect to fundamental eigenfrequency. Int J Adv Struct Eng 2017;9:365–74. doi:10.1007/s40091-017-0172-y.

[6] Karakale V. Use of Structural Steel Frames for Structural Restoration of URM Historical Buildings in Seismic Areas. J Earthq Tsunami 2017;11:1750012. doi:10.1142/S1793431117500129.

[7] AlHamaydeh M, Abdullah S, Hamid A, Mustapha A. Seismic design factors for RC special moment resisting frames in Dubai, UAE. Earthq Eng Eng Vib 2011;10:495–506. doi:10.1007/s11803-011-0084-y.

[8] Asteris PG, Repapis CC, Tsaris AK, Di Trapani F, Cavaleri L. Parameters affecting the fundamental period of infilled RC frame structures. Earthquakes Struct 2015;9:999–1028. doi:10.12989/eas.2015.9.5.999.

[9] Oliveira CS, Navarro M. Fundamental periods of vibration of RC buildings in Portugal from in-situ experimental and numerical techniques. Bull Earthq Eng 2010;8:609–42. doi:10.1007/s10518-009-9162-1.

[10] Asteris PG, Repapis CC, Repapi E V., Cavaleri L. Fundamental period of infilled reinforced concrete frame structures. Struct Infrastruct Eng 2017;13:929–41. doi:10.1080/15732479.2016.1227341.

[11] Aghayari R, Ashrafy M, Tahamouli Roudsari M. Estimation the base shear and fundamental period of low-rise reinforced concrete coupled shear wall structures. Asian J Civ Eng 2017;18:547–66.

[12] Guminiak M. Free vibrations analysis of thin plates by the boundary element method in non-singular approach. Pr Nauk Inst Mat i Inform Politech Częstochowskiej 2007;6:75–90.

[13] Nilesh VP, Desai AN. Effect of height and number floors to natural time period of a multi- storey building. Int J Emerg Technol Adv Eng 2012;2.

[14] Lengvarský P, Bocko J, Hagara M. Modal Analysis of Titan Cantilever Beam Using ANSYS and SolidWorks. Am J Mech Eng 2013;1:271–5. doi:10.12691/ajme-1-7-24.

[15] Christopher Arnold. Chapter 4 - Earthquake Effects on Buildings. Des. Earthquakes A Man. Archit. | FEMA.gov, 2006.

[16] Goel RK, Chopra AK. Period Formulas for Moment-Resisting Frame Buildings. J Struct Eng 1997;123:1454–61. doi:10.1061/(ASCE)0733-9445(1997)123:11(1454).

[17] FEMA. 310-Handbook for seismic evaluation of buildings- A pre-standard. 1998.

[18] IS1893:2002. Part 1: Criteria for earthquake resistant design of structures, general provisions and

buildings. 2002.

[19] BNBC. Bangladesh National Building Code. Hous Build Res Inst 2014;Part 6(2).

[20] Naeim F. Dynamics of structures: Theory and applications to earthquake engineering, 2nd edition. Earthq Spectra 2001;17:549. doi:10.1193/1.1586188.

[21] Beer FP (Ferdinand P. Mechanics of materials. McGraw-Hill Higher Education; 2009.

[22] Sandra Brown. Seismic analysis and shake table modeling: using a shake table for building analysis. University of Southern California; 2007.

[23] Christovasilis IP, Filiatrault A, Wanitkorkul A. Seismic testing of a full-scale wood structure on two shake tables. 14th World Conf. Earthq. Eng., Beijing, China: 2008.

[24] Tiziano S, Daniele C, Roberto T, Maurizio P. Shake table test on 3-storey light-frame timber building. World Conf. Timber Eng., Auckland, No. 77, I-38123: 2012.

[25] Kim S-E, Lee D-H, Ngo-Huu C. Shaking table tests of a two-story unbraced steel frame. J Constr Steel Res 2007;63:412–21. doi:10.1016/j.jcsr.2006.04.009.

[26] Siddika A, Awall MR, Mamun MA Al, Humyra T. Free vibration analysis of steel framed structures. J Rehabil Civ Eng 2019;7:70–8. doi:10.22075/jrce.2018.12830.1224.

[27] Ghaffarzadeh H, Talebian N, Kohandel R. Seismic demand evaluation of medium ductility RC moment frames using nonlinear procedures. Earthq Eng Eng Vib 2013;12:399–409. doi:10.1007/s11803-013-0181-1.

[28] Srinivasan R, Suresh Babu S, Itti S V. A study on performance of 3D RC frames with masonry in-fill under dynamic loading conditions. KSCE J Civ Eng 2017;21:322–8. doi:10.1007/s12205-016-0537-y.

[29] Liu S, Warn GP, Berman JW. Estimating Natural Periods of Steel Plate Shear Wall Frames. J Struct Eng 2013;139:155–61. doi:10.1061/(ASCE)ST.1943-541X.0000610.

[30] Tomasiello S. A Simplified Quadrature Element Method to compute the natural frequencies of multispan beams and frame structures. Mech Res Commun 2011;38:300–4. doi:10.1016/j.mechrescom.2011.04.002.

[31] Zalka K. A simplified method for calculation of the natural frequencies of wall–frame buildings. Eng Struct 2001;23:1544–55. doi:10.1016/S0141-0296(01)00053-0.