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Design Formula at Ultimate Stresses for Ekki (Lophira Alata) Timber Beam

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ABSTRACT

The global pursuance of design method for economic utilization of building material is the basis of this study. The ultimate stress design method proposed in this study is in line with this pursuit as it encourages full utilization of material section and thus brings in economy in material application. This is the focus of this study on appropriate design procedure for Ekki (Lophira alata) timber beam. This new method can replace the conventional design which is based only on modulus of rupture that has been reported not a rational method because it relies only on the extreme thin fibre tensile strength of the beam in flexure, whereas, for such a beam, it is subjected to both tension stress (below the neutral axis) and compression stress (above the neutral axis). Based on this fact, the method generated two stress expressions, one for axial tension and the other for axial compression, represented by their typical stress-strain equations for loaded Ekki specimens. To develop the beam design equations, the axial typical stress-strain equations were converted to bending expressions of the stress-neutral axis depth relationships for the Ekki beam section. These relationships were further simplified and tested to give theoretical results that compare well with the experimental values.

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1. Introduction

The timber material of interest in this report is Ekki wood. It is a tree found locally in lowland rain forest of Nigeria and also in the wetter past of rain forest of other west Africa countries. It is locally called (in Nigeria) eba-(Bini),akuifo-(ibo) and in other countries it is called azobe(Ivory coast) , bongossi (Cameroon) and kaku (Ghana), red ironwood (BW Africa). Its botanical name is Lophira alata and belongs to the family of Ochnaceae. Ekki is used as wagon planks, heavy construction purposes, dock and river piling, wharfs and heavy duty flooring [1–3]. Some of these structural uses are composed of beam structural elements for which the theory developed in this study can be applied. Generally, timber strength properties are in three dimension in the stem: radial, tangential and longitudinal [4]. Each of these properties are important for some specific structural design. For example in beams, flexural stress occurs, where stresses are tensile below the neutral axis and compressive above. Both tensile and compressive stresses occur in the longitudinal directions of the beam.

However, there has been no established design procedure for Ekki timber beam, which can be used for its rational application. A conventional design information on wood beam is popularly used and it is based on the use of modulus of rupture, given by [4,5], as

$$\sigma = M/Z \qquad (1)$$

where σ is the tensile stress (called modulus of rupture) at the bottom face of the beam, M is the applied moment and Z is the section modulus. Eqn. (1) is reported not reliable, long abolished for materials like steel, concrete, pre-stressed concrete and it survives only in the testing of timber. The equation is reported as not having practical value and cannot be used to estimate the ultimate bending capacity of beams [6]. It is on this premise that alternative design method is being sought.

The alternative method proposed was application of material tensile and compressive stress-strain for the design of a timber beam [6]. This method is what is being developed for the loaded Ekki. To accomplish similar theory for the design of Ekki timber beam, the stress and strain data of Ekki timber are obtained and used to derive typical stress-strain equations in tension and in compression. These equations are converted to stress versus neutral axis depth relationships of timber beam section and further analysed to obtain the beam design equations. Similar theory has been developed for Ayin (Anogeissus leiocarpus) beam [7].

2. Theory

2.1. Assumptions

In order to accomplish the derivation of the design formula, the following assumptions were made:

(i) The distribution of strains is compatible with the distorted shape of the cross-section, with the strain line above or below the neutral axis being linear. This assumption of linearity of strain in a timber beam section is appropriate and which also is applied in concrete beams [4–6].

(ii) At ultimate, the distribution of stress is not linear with the distorted shape of the cross-section, this also is assumed in [6].

(ii) The moment of resistance developed by the section must balance the moment caused by the applied loads, for static equilibrium also assumed in [6,8].

2.2. Analysis of beam section

Analysis of the beam section is carried out using the schematic diagram of Ekki beam loaded in flexure shown in Figure 1a. Also shown with this are Figures 1b ,1c,1d and 1e, that stand respectively for section x – x of the beam, the polynomial stress diagram, the strain diagram and equivalent rectangular stress diagrams. These figures contain parameters required to develop the beam formula and are defined as follows:

(a) Neutral axis (N.A.) depth for tension and compression stresses (x_t, x_c)

Neutral axis is the distance from the outer-most face of the beam to the neutral axis as shown in Figure 1d and it is represented by x_t for tension stress zone and x_c for compression stress zone. When x_c is represented by x, then, the relationship between this and overall-depth d of the beam is given by $x_t = d-x$

(b) Tensile and compressive stresses (f_t , f_c)

These are stresses across the section presented in Figure 1c and the maximum (ultimate) value is at the extreme fibre level and zero value at the neutral axis. The ultimate tensile and compressive stresses are represented by f_{tu} , f_{cu} , respectively.

(c) Tensile and compressive strains (ε_t , ε_c)

These are the strains across the section shown in Figure 1d. The maximum strain value occurs at the extreme fibre level and zero at the neutral axis. The ultimate tensile and compressive strains are represented by ε_{tu} and ε_{cu} respectively. From the figure, since the strain line is linear, the slope of the strain is constant and denoted by k as

$$k_t = \frac{\varepsilon_{tu}}{x_t} \tag{2}$$

for tension, and

$$k_c = \frac{\varepsilon_{cu}}{x_c} \tag{3}$$

for compression zone.

From eqn (2) and (3), the strain at depth x_t or x_c from neutral axis level is

$$\varepsilon_t = k_t x_t \tag{4}$$

at the tension zone, and ,

$$\varepsilon_c = k_c x_c \tag{5},$$

at the compression zone.

(d) Resultant tensile and compressive forces (F_t , F_c)

They are obtained by integrating stresses f_t , f_c , respectively over the stressed areas on either side of N.A. in Figure 1c. Thus, resultant tensile force F_t , using the method of area integration analysis method given in reference [9], is denoted by

$$F_t = \int_{x_t = 0}^{x_t} f_t dx_t \tag{6}$$

and

likewise, the resultant compressive force F_c is given by

$$F_c = \int_{x_c = 0}^{x_c} f_c dx_c \tag{7}$$

The resultant forces act in one direction along the longitudinal axis of the beam.

(e) Expression for centroidal distance for resultant tensile and compressive forces ($\bar{x}_t, \bar{x}_c$)

Centroidal distance is the distance from the resultant force to the neutral axis. They are shown in Figure 1c and are obtained when the sum of moment of elemental stresses about the neutral axis in the tension or compression area is divided by the resultant force. That is,

$$\bar{X}_t = \frac{\int f_t x_t dx_t}{\int f_t dx_t} \tag{8}$$

for tension and similarly, that of compression is

$$\bar{X}_c = \frac{\int f_c x_c dx_c}{\int f_c dx_c} \tag{9}$$

(f) Lever arm Z

This is the distance between centroids of the two resultant tension and compression forces presented in Figure 1c and it is represented by Z, given as

$Z = \bar{x}_t + \bar{x}_c$ (10)

(g) Mean tensile and compressive stresses ($\bar{f}_t, \bar{f}_c$)

They are shown in Figure 1e and are obtained when the resultant force is divided by the neutral axis depth, that is,

$$\bar{f}_t = \frac{\int_{x_t=0}^{x_t} f_t dx_t}{x_t} \tag{11}$$

for tension, and

$$\bar{f}_c = \frac{\int_{x_c=0}^{x_c} f_c dx_c}{x_c} \tag{12}$$

for compression

In order to solve eqns. (2 to 12), and develop the stress-strain equations, the data for strains and stresses, that is ε_{tu} , ε_{cu}, f_{tu} and f_{cu} **,** are required. These therefore prompted the experimental work carried out on Ekki timber material.

3. Experimental work

Laboratory tests were carried out on 50 specimens, as directed in [10], of Ekki wood material, for direct axial tensile and direct compressive stresses parallel to grain, respectively, following the BS 373 requirement [11]. The Ekki material used for the experimental work was the matured air-dried pieces, obtained from a timber market in Ilorin, Nigeria. They were cut and shaped to the required specimen sizes and tested using a Testometric Universal Testng Machine model M500 of 50KN capacity, obtained in the Faculty of Engineering University of Ilorin, Nigeria. The specimen nominal dimensions for tensile test was 50 mm long and 6 mm square shape at the gauge section and the specimen for the compressive test was uniform being 100mm long and 20 by 20 mm in section. Each specimen was tested to failure. Material properties were determined in accordance with [12,13]

(a) Experimental data

Experimental data are:

* ultimate axial tensile stress ranged from 100 N/mm^2 to 240 N/mm^2 with an average of 143 N/mm^2.

* the strain corresponding to ultimate tensile stress: ranged from 0.2 % to 4.1% with an average of 0.0221;

*the experimental ultimate axial compressive stresses ranged from 57 N/mm^2 to 102 N/mm^2 with an average of 73 N/mm^2

• the experimental compression strain: ranged from 1.96% to 3.43% with an average of 2.85 %

* At the peak of the ultimate compression stress-strain curve the optimum stress is 67.53 N/mm^2 with a corresponding strain of 0.026

*The average Ekki material density is 1.025kg/m^3 (ranging from between 0.97 to 1.10 kg/m^3) and the moisture content of material at test ranged from 9 to 12 % with an average of 10.2 %.

(b) Typical Stress – Strain Equations

The experimental stress-strain results were obtained from zero load to failure of each specimen and were analysed to obtain the stress-strain regression equation for the specimen using the Microsoft Excel Word 2003 and cubic polynomial equation of the form $ax+ax^2+ ax^3$. Average values of the coefficients (a, b and c) in the equations for all the fifty specimens were obtained and used as the typical short term stress strain equation. These equations are

$$\sigma_t = 6570\varepsilon - 202604\varepsilon^2 + 9000000\varepsilon^3 \quad (13)$$

For tension; and,

$$\sigma_c = -1367\varepsilon + 386469\varepsilon^2 - 9000000\varepsilon^3 \quad (14)$$

for compression

where σ and ε denote stress and strain respectively. The R^2 values ranged from 0.75 to 0.96

4. Derivation of beam design parameters and equations.

4.1. Determination of stress –neutral axis depth expression

In the stress–strain eqns (13 and 14), substitution was made into them for the expression for strain $\varepsilon=kx$ (given in eqn. 4 and 5), to obtain the expression relating the flexural stress and neutral axis depth given as

$$f_t = 9000\,000\,(k_t x_t)^3 - 202604(k_t x_t)^2 + 6570\,(k_t x_t) \text{ in tension} \quad (15)$$

$$f_c = -9\,000000\,(k_c x_c)^3 + 386469\,(k_c x_c)^2 - 1367(k_c x_c) \text{ in compression} \quad (16)$$

When equations (15 and 16), are multiplied by x, they become

$f_t x_t = 9000\,000\ k_t^3 x_t^4 - 202604\ k_t^2 x_t^3 + 6570\ k_t x_t^2$ in tension (17)

$f_c x_c = -9\,000000\ k_c^3 x_c^4 + 386469\ k_c^2 x_c^3 + 1367\ k_c x_c^2$ in compression (18)

4.2 Derivation of design parameters

Design parameters were obtained by integration and evaluation of the stress-neutral axis depth expressions (15-18) using the limits of x_t and x_c , and the expression for k, given as: $0 \leq x_c \geq x$; $0 \leq x_t \geq d\text{-}x$; $k_c = \varepsilon_c / x = 0.026/x$; and $k_t = \varepsilon_t / (d\text{-}x) = 0.022/(d\text{-}x)$). The substitution of the solutions of the integrations into eqn. (6-12), and solving them in terms of width b , depth d and experimental ultimate stresses f_{tu} and f_{cu} , using balanced failure design method (i.e $F_t = F_c$) as adopted for concrete beam section in a literature [8], yielded the following equations:

x (or x_c) $=0.49d$ (19)

$x_t = 0.51d$ (20)

$\bar{f}_t = 0.444 f_{tu}$ (21)

$\bar{f}_c = 0.97 f_{cu}$ (22)

$F_t = 0.2264 f_t bd$ (23)

$F_c = 0.4753 f_c bd$ (24)

$\bar{x}_t = 0.34d$ (25)

$\bar{x}_c = 0.34d$ (26)

$Z = 0.68d$ (27)

Where the parameters are as previously defined.

4.3. Design equations for Ekki beam

This is derived using the moment of resistance M_R of the beam section, given as

$M_R = F_c Z$ or $F_t Z$

After substitution for F and Z ,

$M_{Rt} = 0.154 f_{tu} bd^2$ (28)

in term of tension stress ; and,

$M_{Rc} = 0.3232 f_{cu} bd^2$ (29)

in term of compression stress.

Therefore, eqns. (28 and 29) are the design formula for the Ekki beam. These two expressions are applicable and the one that gives the lesser value is the applicable design formula.

5. Validity of the design equations

This was carried out by comparing them with the conventional formula (equation 1). In the conventional formula, the elastic modulus Z is 1/6 (or 0.167) and the plastic modulus is 0.25 [14]. Thus, ultimate moment of resistance $Mu=0.167f_tbd^2$ and $Mu=0.257f_tbd^2$ respectively. In this study, $Mu=0.154f_tbd^2$ in term of tensile strength and $M_u = 0.32f_cbd^2$ in term of compressive strength. Therefore, the coefficients of these expressions are comparable, but their values will vary depending on the modulus of rupture, tensile or compressive stresses respectively.

The second validation carried out was application of the design eqns. (28 and 29) on experimental load result on nine Ekki beams. The results are shown in Tables 1 and 2. From the tables, both tension and compression design equations produced similar results that varied between 66% and 87 % of the experimental values.

6. Discussion

The design eqns. (28 and 29) can be used to design Ekki beam of rectangular cross section of width b and depth d. It is the experimental average ultimate compressive or tensile strength that is required for substitution in the respective equation. Also, the while flexural stress is on thin outermost layer, the tensile and compressive stresses cut across the whole surface of the specimen. The complex equations that were obtained from the analysis of polynomial stress distribution (in Figure 1c) have been replaced with the simple relationship developed. The eventual design equations obtained can be used to predict the load carrying capacity of Ekki beam, with accuracy varying from 66 to 87%. In order to avoid large variation in result, the Ekki timber material must be from a mature tree, air-dried and from the heartwood part of the stem. Further design strength can be used for tensile and compressive stresses in the equation 28 and 29. This strength is the basic stress, usually used to take care of large variation in result [15]. It is given as

$$Basic\ stress = \frac{mean - 2.33S}{2.25\ or\ 1.4} \tag{30}$$

Where 2.25 is used for tension parallel to grain and 1.4 is used for compression parallel to grain and S is the standard deviation. Therefore to take care of the large variation in result, the basic stress is substituted in equations 28 and 29.

7. Conclusion

From the analysis, the conclusion can be drawn as follows:

The mean tensile strength in the Ekki beam section is $0.444f_{tu}$ and the resultant tensile force F_t in the section is $0.2264f_{tu}$ bd. The moment of resistance (design formula) in terms of tensile strength is $M_{Rt} = 0.154f_{tu}\, bd^2$ where b is the beam width, d is the overall depth and f_{tu} is the ultimate tensile stress with a value of 143 N/mm^2 .

The mean compressive strength in the beam section is $0.97f_{cu}$ and the resultant compressive force F_c in the section is $0.4753f_{cu}bd$. The lever-arm which is the distance between the centroids of the resultant tensile and compressive forces is 0.68d. The moment of resistance (design formula) in terms of compressive strength is $M_{Rc} = 0.3232f_{cu}bd^2$, where f_{cu} is the ultimate compressive stress having a value of 68 N/mm^2

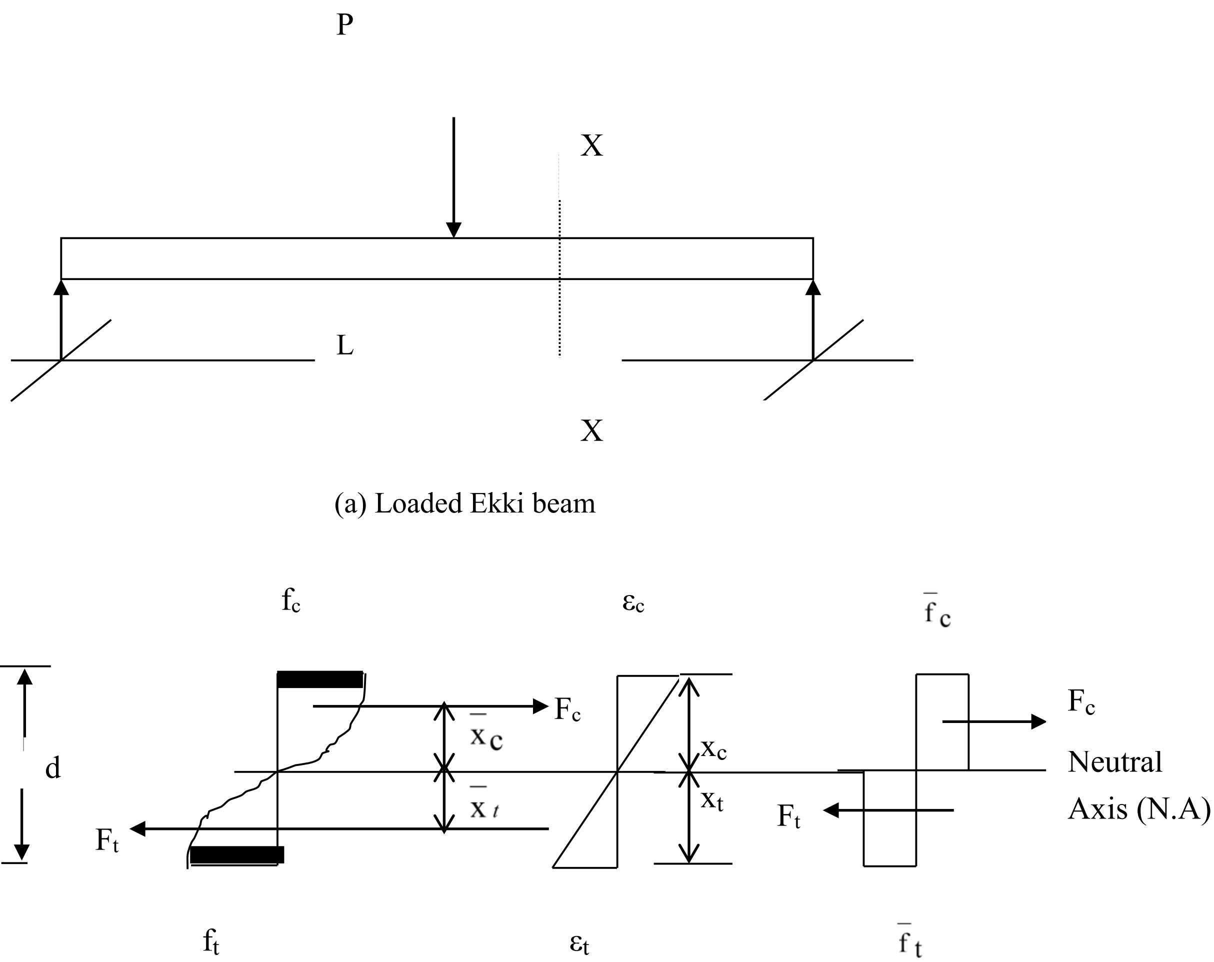
The two design formula ($M_{Rt} = 0.154\, f_{tu}\, bd^2$ and $M_{Rc} = 0.3232f_{cu}bd^2$) can be used to design the beam section.

Table 1
Comparison between experimental and designed (predicted) ultimate load on air-dried Ekki beam loaded in flexure using tension eqn. (32) (f_{tu}=143 N/mm^2).

Specimen No.	Width b mm	Depth d (mm)	Span L (mm)	Exp failure Load (kN) (centrally applied)	Predicted mom. of resistance $M_R = 0.154f_{tu}\, bd^2$ (Nmm)	Predicted Load= $(4M_R)/L$ (N)	Predicted /Exp. Load (col.7/col..5)
	2	3	4	5	6	7	8
1	14.0	13.5	160	1718.5	56189.13	1404.73	0.82
2	13.8	13.0	160	1681.3	51359.71	1283.99	0.76
3	14.0	13.0	160	1646.5	52104.05	1302.60	0.79
4	14.0	12.5	160	1695.1	48173.13	1204.33	0.71
5	14.0	13.0	160	1505.8	52104.05	1302.60	0.87
6	14.0	13.0	160	1977.6	52104.05	1302.60	0.66
7	13.0	13.0	160	1494.0	48382.33	1209.56	0.81
8	13.8	13.5	160	1593.4	55386.43	1384.66	0.87
9	14.0	13.5	160	1872.7	56189.13	1404.73	0.75
						average	0.78

Table 2
Comparison between experimental and designed (predicted) ultimate load on air-dried Ekki beam loaded in flexure using compression eqn. (33) f_{cu}=68.0 Nmm^2.

Specimen No.	Width b mm	Depth d (mm)	Span L (mm)	Exp failure Load (kN) (centrally applied)	Predicted mom. of resistance $M_R = 0.3232f_{cu}bd^2$ (Nmm)	Predicted Load= $(4M_R)/L$ (N)	Predicted /Exp. Load (col.7/col..5)
	2	3	4	5	6	7	8
1	14.0	13.5	160	1718.5	56075.85	1401.90	0.82
2	13.8	13.0	160	1681.3	51256.16	1281.40	0.76
3	14.0	13.0	160	1646.5	51999.00	1299.98	0.79
4	14.0	12.5	160	1695.1	48076.00	1201.90	0.71
5	14.0	13.0	160	1505.8	51999.00	1299.98	0.86
6	14.0	13.0	160	1977.6	51999.00	1299.98	0.66
7	13.0	13.0	160	1494.0	48284.79	1207.12	0.81
8	13.8	13.5	160	1593.4	55274.76	1381.87	0.87
9	14.0	13.5	160	1872.7	56075.85	1401.90	0.75
						Average	0.78



(b) Section X-X (c) Polynomial stress diagram (d) Strain diagram (e) Equivalent rectangular diagram

Fig. 1. (a) Loaded Ekki beam , (b) section , (c) stress and (d) strain diagrams.

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