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Buckling of Tapered Columns with Polygon Cross-Section

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ABSTRACT

This paper is concerned with the elastic stability of slender tapered columns of regular polygon cross-section with constant volume is presented. Various end conditions of the tapered columns such as pinned ends, clamped -pinned ends and clamped ends are considered in this paper. The analysis is developed using finite element method for linear, parabolic and sinusoidal tapers assuming firstly circular cross-section. The polygon cross-sections other the circular one are studied by investigating a direct relation between the side number of the polygon cross-section and the critical load parameter. The main parameter in this study is the section ratio that defined as the ratio between the section depths at the mid-span of the considered column to that at the column ends. The obtained numerical results are introduced in many curves to describe the relation between the buckling load and the section ratio considering linear, parabolic and sinusoidal tapers and various side number of the polygon cross-section for each end conditions. The obtained results of the buckling factor are simple with satisfactory accuracy compared with results that obtained in other theoretical studies.

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1. Introduction

Stability of columns is an important item in structural engineering so that the critical buckling loads of columns and its behavior's under different cases of loading and different cases of end conditions have been studied and documented in many researches and texts such as Timoshenko [1]. The main goals of the design are the safety and economic so that increasing of the critical loads of specific span length columns without increasing the column volume, are important in the viewpoint of optimal design. Also, for architecture proposes, tapered columns may be required.

Many studies for columns with constant volume are carried out by Keller [2], Tadjbakhsh and Kelller [3], Barnes [4] and Cox and Overton [5] to determine the critical load and the strongest columns. Lee and Oh [6] developed Numerical methods to solve elastic and critical buckling loads of simply supported tapered column with constant volume. Also, Lee, Oh and Li [7] studied the stability of tapered columns with polygon cross-section considering the case of clamped ends only. The differential equation governing the buckled shape of solid tapered columns with polygon cross-section having a constant volume is derived by Lee, Karr and Kim [8] that solved numerically.

In this paper, stability study of slender solid tapered columns having constant volume with regular polygon cross-section is carried out using finite element method assuming linear, parabolic and sinusoidal tapers and considering various end conditions. Also, the effect of the number of sides of the cross-section in the critical load is introduced in a direct relation.

2. Object column

Consider a slender solid tapered column with specific span length L and constant volume V as shown in Figure 1. The cross-section is a regular polygon and its depth h varies with the coordinate x. The cross-section depth at the column ends and at the mid-span are h_0 and h_m respectively.

The cross-sectional area A and the moment of inertia I of the regular polygon cross-section are as follows

$$A = \frac{m}{2}\sin\left(\frac{2\pi}{m}\right)h^2 \tag{1}$$

$$I = \frac{m}{4}\sin\left(\frac{\pi}{m}\right)\cos^3\left(\frac{\pi}{m}\right)\left[1+\frac{1}{3}\tan^2\left(\frac{\pi}{m}\right)\right]h^4 \tag{2}$$

Where h is the cross-sectional depth at a distance x from the column end.

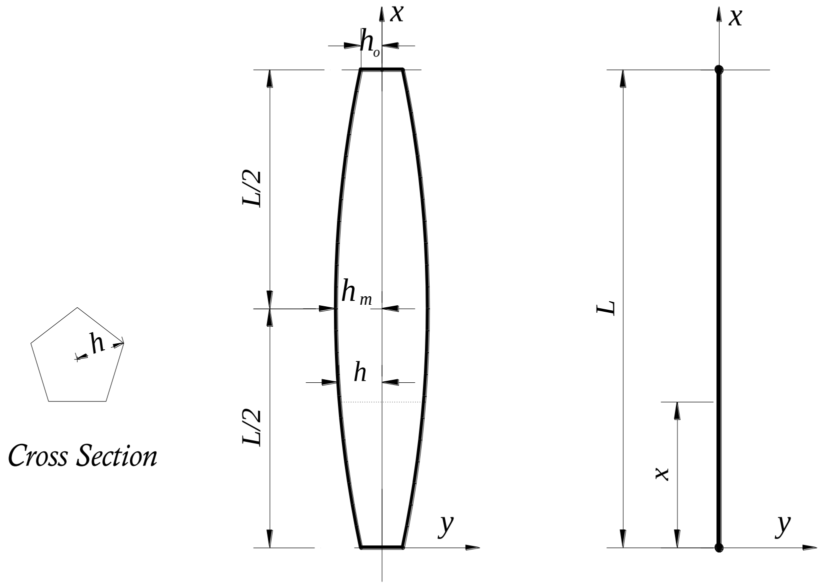


Fig. 1. Tapered column with regular polygon cross-section.

For linear taper, the variable depth h as a function of x is determined as follows

$$h=\left[1+2(n-1)\left(\frac{x}{L}\right)\right]h_0 \qquad 0\le x\le L/2 \tag{3-a}$$

$$h=\left[1+2(n-1)-2(n-1)\left(\frac{x}{L}\right)\right]h_0 \qquad L/2\le x\le L \tag{3-b}$$

Where, the section ratio n is a non-dimensional parameter as follows

$$n=\frac{h_m}{h_0} \tag{4}$$

Then, the volume V can be determined as follows

$$V=\int_0^L A dx = m\sin\left(\frac{\pi}{m}\right)\cos\left(\frac{\pi}{m}\right)\beta L h_0^2, \quad \beta=\frac{1}{3}\left(n^2+n+1\right) \tag{5}$$

Second, for parabolic taper, the cross-section depth h and volume V are obtained as follows

$$h=\left[1+4(n-1)\left(\frac{x}{L}\right)-4(n-1)\left(\frac{x}{L}\right)^2\right]h_0 \tag{6}$$

$$V=\int_0^L A dx = m\sin\left(\frac{\pi}{m}\right)\cos\left(\frac{\pi}{m}\right)\beta L h_0^2, \quad \beta=\frac{1}{15}\left(8n^2+4n+3\right) \tag{7}$$

Finally, for the sinusoidal taper, the cross-section depth h and volume V are obtained as follows

$$h=\left[1+(n-1)\sin\left(\frac{\pi x}{L}\right)\right]h_0 \tag{8}$$

$$V=\int_0^L A dx = m\sin\left(\frac{\pi}{m}\right)\cos\left(\frac{\pi}{m}\right)\beta L h_0^2, \quad \beta=\frac{1}{2}(n-1)^2+\frac{4}{\pi}(n-1)+1 \tag{9}$$

Generally, Eqs (3, 6 and 8) can be written in the following form

$$h=\alpha h_0 \tag{10}$$

Where α is a function of n and x

Then,

$$V=m\sin\left(\frac{\pi}{m}\right)\cos\left(\frac{\pi}{m}\right)\frac{\beta L}{\alpha^2}h^2 \tag{11}$$

Substituting Eq. (11) in Eq. (2)

$$I=\frac{1}{m}\left[\cot\left(\frac{\pi}{m}\right)+\frac{1}{3}\tan\left(\frac{\pi}{m}\right)\right]\left[\frac{V^2\alpha^4}{4\beta^2L^2}\right] \tag{12}$$

Assuming a column having a uniform circular cross-section with volume V .The moment of inertia of its cross-sectional area I_e can be easily determined as follows

$$I_e=\frac{V^2}{4\pi L^2} \tag{13}$$

Substituting Eq. (13) in Eq. (12)

$$I=\frac{\pi}{m}\left[\cot\left(\frac{\pi}{m}\right)+\frac{1}{3}\tan\left(\frac{\pi}{m}\right)\right]\left[\frac{\alpha^4}{\beta^2}\right]I_e \tag{14}$$

Thus, the critical buckling load for columns under study can be assumed as

$$P_{cr}=\phi\frac{\pi^2 E I_e}{L^2}, \quad \phi=\phi_1.\phi_2 \tag{15}$$

Where,

E is the modulus of elasticity of the column material,

ϕ is the buckling load parameter

ϕ_1 represents the effect of the side number of the polygon section as cleared above. This factor doesn't depend on the type of the taper (linear, parabolic, or sinusoidal) and can be expressed as follows:

$$\phi_1 = \frac{\pi}{m}\left[\cot\left(\frac{\pi}{m}\right) + \frac{1}{3}\tan\left(\frac{\pi}{m}\right)\right] \tag{16}$$

But the factor ϕ_2 represents the effect of taper type and the section ratio n that will be obtained numerically by finite element method on the next section.

The factor ϕ_1 can be determined as shown in Table 1

Table 1.
Effect of the side number of the polygon section m in buckling factor.

m	3	4	5	8	∞
ϕ_1	1.209	1.047	1.017	1.002	1.000

3. Method and assumptions

The column under study is modeled as a solid element as shown in Figure 2 with circular cross-section ($m = \infty$) where the effect of the side number of the polygon section can be determined as discussed by Eq. (16) and Table 1. The column length is divided by twenty segments and the diameter of each segment cross-section is calculated by using Eqs (3, 6 and 8) for linear, parabolic and sinusoidal tapered columns with different values of section ratio n. Different end conditions such as pinned ends, clamped-pinned, ends and clamped ends are considered in the models under study. The critical buckling loads for the column under study have been obtained using SAP2000 program based on the finite element method.

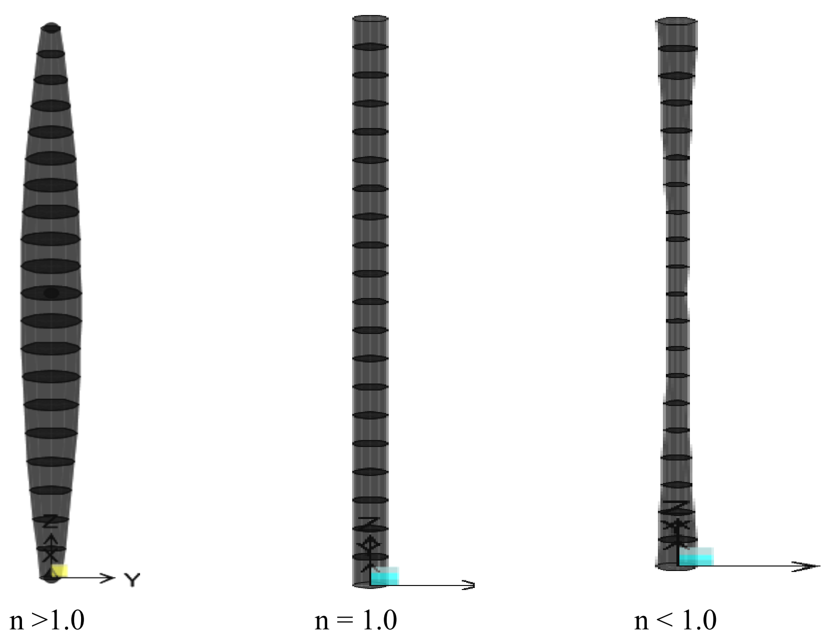
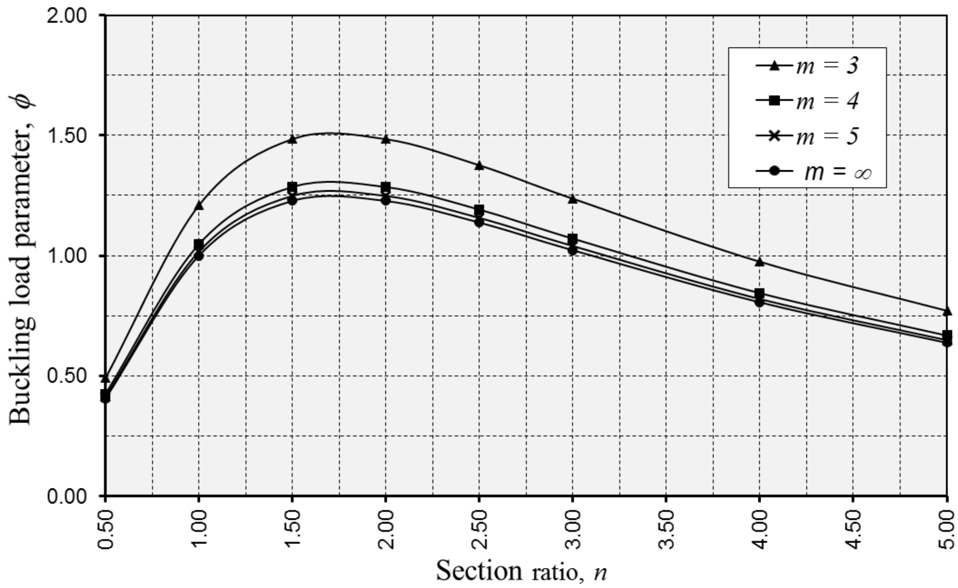


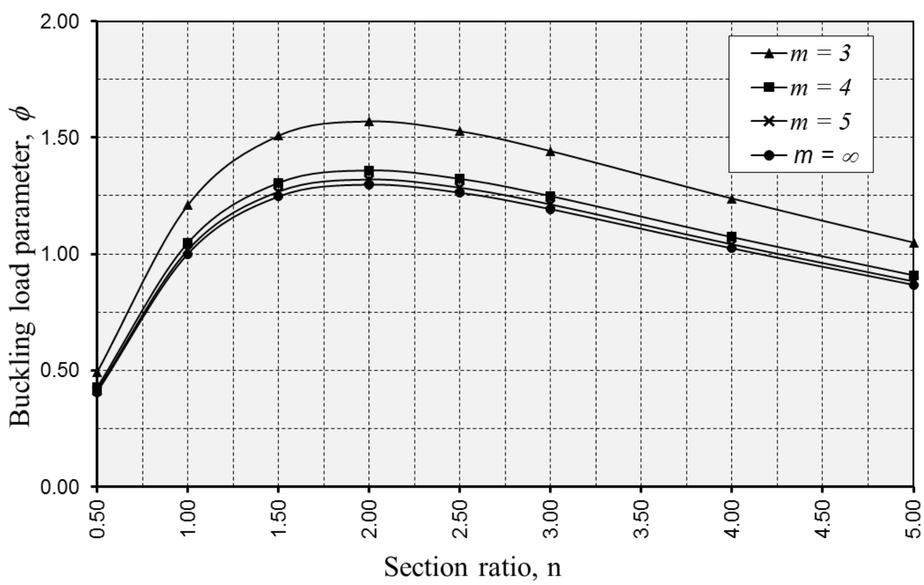
Fig. 2. Finite element model of the Column under study.

4. Results and discussions

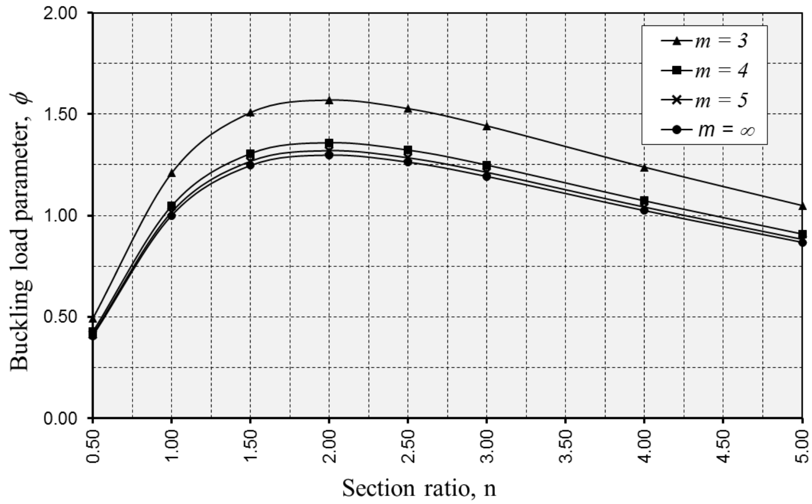
Figures 3 shows the relation between buckling factor ϕ and the section ratio n of pinned ends column for linear, parabolic and sinusoidal tapers considering various number of the cross-section sides (m =3, 4, 5 and ∞). Also, Figures 4 and 5 describe the same relations with other end conditions such as clamped -pinned ends and clamped ends.



a) Linear tapered column

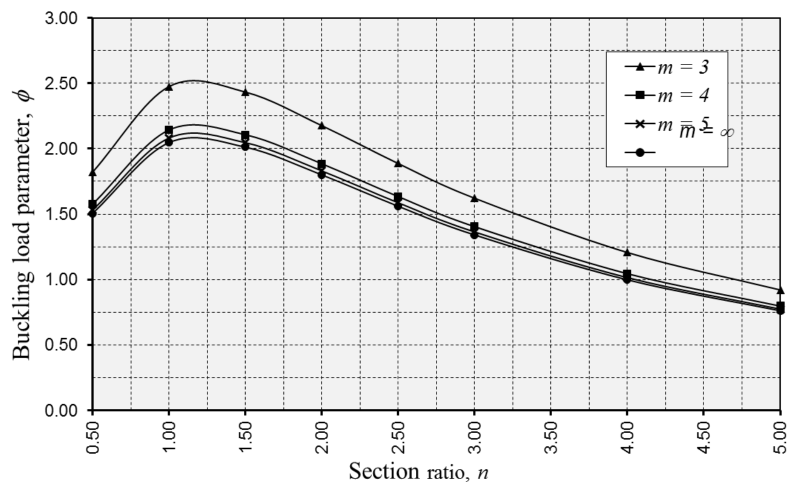


b) Parabolic tapered column

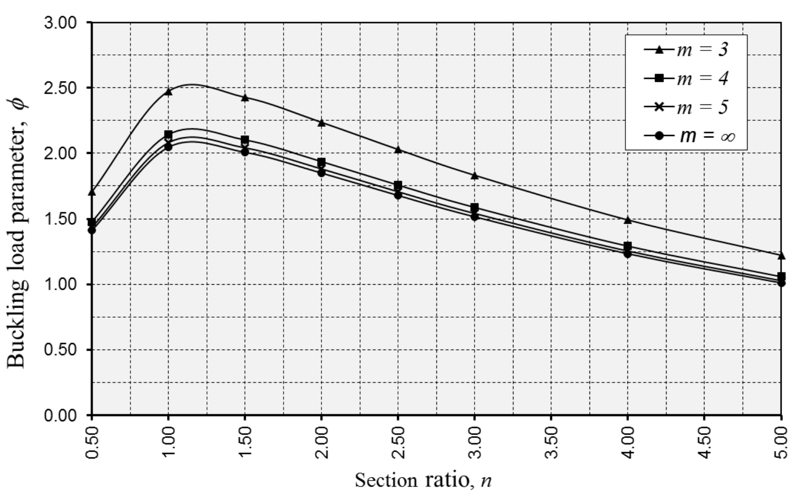


c) Sinusoidal tapered column

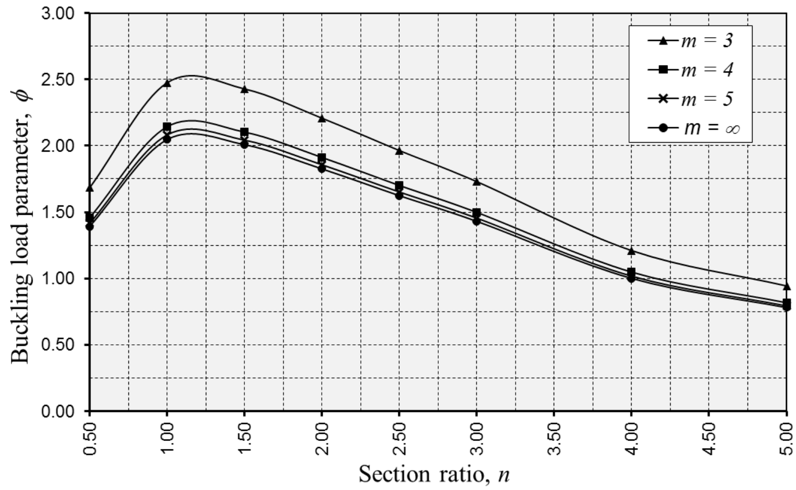
Fig. 3. Buckling load parameter of tapered columns with pinned ends.



a) Linear tapered column

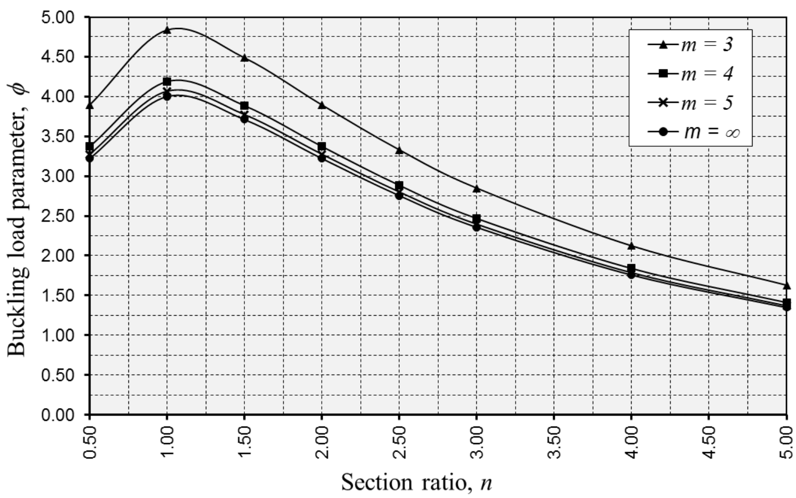


b) Parabolic tapered column

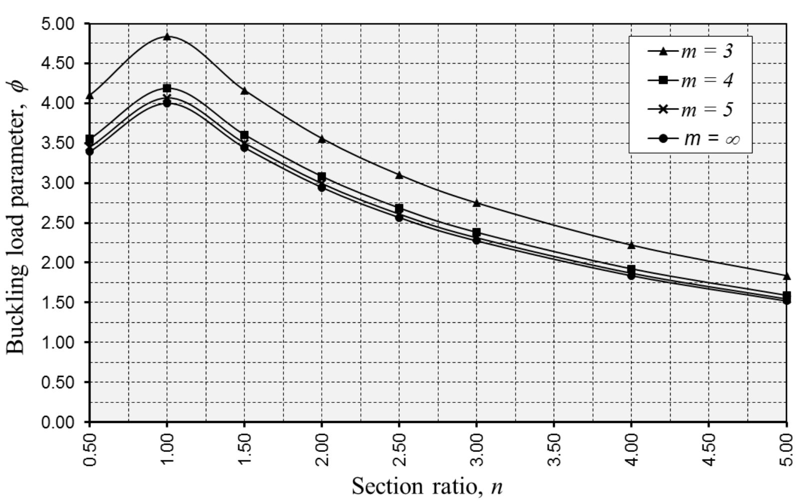


c) Sinusoidal tapered column

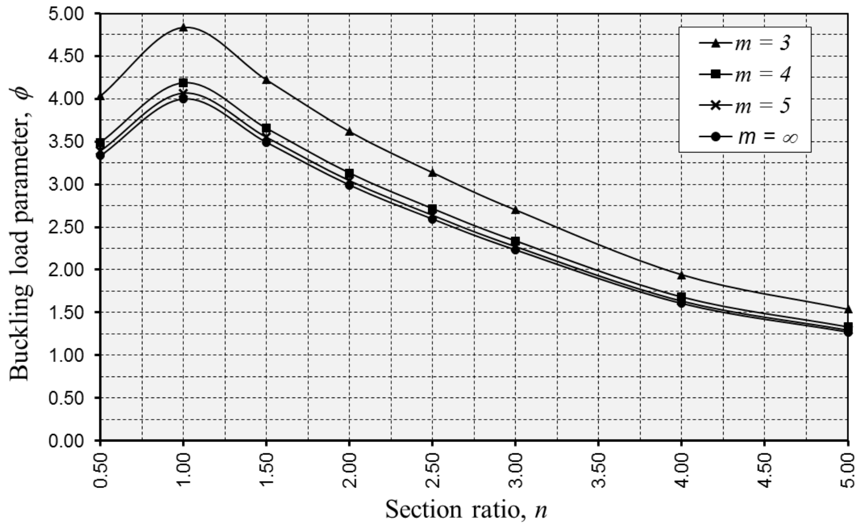
Fig. 4. Buckling load parameter of tapered columns with clamped-pinned ends.



a) Linear tapered column



b) Parabolic tapered column



c) Sinusoidal tapered column

Fig. 5. Buckling load parameter of tapered columns with clamped ends.

All these figures illustrate the effect of section ratio n, side number m, taper type and the end conditions to the buckling coefficient ϕ.

The most resistant columns to buckle can be obtained by reading the peak points of the resulted curves that represent the maximum critical buckling parameter corresponding to the taper type for each end conditions.

It is obvious that at the columns with uniform circular section (n =1 and $m = \infty$), the buckling parameter equal to the exact factors of the known cases for various end conditions.

From the Figures 3 to 5 and Table 1, it is noticed that the buckling parameter are largest at triangular cross-section (m=3) and smallest at circular cross-section ($m = \infty$). Also, it is noticed that the resulted buckling loads are very close if the side number m equal to or greater than 4 ($m \geq 4$).

Table 2 shows a comparison between the results of the critical buckling loads from this study and that obtained by Lee, Karr and Kim [8] for various end conditions, different taper types and several side numbers. The chosen values of the section ratio n are the values corresponding to the strongest columns. This comparison clears the accuracy of the developed method in this paper that can be easily used by design engineers.

Table 2.
Comparison of buckling load parameter results between this study and reference [8].

Taper type	End conditions	m	n	ϕ This study	ϕ Reference [8]	% Diff.
Parabolic taper	Clamped ends	3	0.836	4.938	4.929	0.190%
		4		4.277	4.269	0.182%
		5		4.153	4.146	0.176%
		∞		4.084	4.076	0.196%
	Clamped-pinned ends	3	1.174	2.495	2.503	-0.336%
		4		2.160	2.167	-0.306%
		5		2.098	2.105	-0.332%
		∞		2.063	2.070	-0.338%
	Pinned ends	3	1.973	1.567	1.572	-0.310%
		4		1.357	1.362	-0.355%
		5		1.318	1.322	-0.303%
		∞		1.296	1.300	-0.308%
Sinusoidal taper	Clamped ends	3	0.854	4.909	4.904	0.109%
		4		4.252	4.247	0.109%
		5		4.129	4.125	0.095%
		∞		4.060	4.056	0.099%
	Clamped-pinned ends	3	1.183	2.499	2.507	-0.312%
		4		2.164	2.171	-0.306%
		5		2.102	2.108	-0.290%
		∞		2.067	2.073	-0.299%
	Pinned ends	3	1.850	1.556	1.558	-0.113%
		4		1.348	1.349	-0.093%
		5		1.309	1.310	-0.088%
		∞		1.287	1.288	-0.078%

5. Conclusions

Finite element method is established in this study for estimating the critical load of solid tapered columns having polygon cross-section with constant volume. Different end constraints such as pinned ends, clamped ends and pinned-clamped ends are studied in this paper. The linear, parabolic and sinusoidal tapers are considered in this study.

The results obtained from this study are introduced in many curves to describe the relation between the critical load and the section ratio *n* for different types of tapers. The obtained curves cover the different end conditions and different side number *m*. Also, the relation between the buckling load coefficient and the side number *m* of the uniform polygon cross-section is derived in closed-form relation.

For each end conditions and appropriate taper type of the column under study, the peak points of the resulted curves represent the strongest column corresponding to the side number.

This present method is simple to use by engineers designing columns under study with satisfactory accuracy compared with results that obtained in published references.

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