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Comparative Three-Dimensional Finite Element Analysis of Piled Raft Foundations

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ABSTRACT

3-dimensional finite element method as a general method to solve complex problems is one of the most powerful numerical methods which can be used for piled raft foundation analysis. These models can consider the complex interaction between soil and structure. Among available 3D FEM' software for modelling pield raft foundations, in this paper MIDAS GTS is used due to its various element type and modeling abilities. In this article, different pile modeling techniques in MIDAS GTS software (like pile modeling by solid elements, modeling by beam elements connected to soil elements and modeling by EPM) are compared with a real pile loading test data. Results showed that all three methods have excellent compatibility with the results of loading test in the linear area of the load-settlement curve, and SEM^r and EPM kept their conformity further in the non-linear area as well. One of the most critical problems in 3D FEM modeling process of piled raft foundations with SEM was an increase in the number of elements when the number of piles increases and that leads to model's slowness and convergence problem. Piles modeling by EPM needs much lower elements; using this method, skin friction resistance, tip resistance and displacement between pile and soil can be easily calibrated with a pile loading test data which facilitates piled raft analysis with a large number of piles. After comparing different pile modeling techniques through MIDAS GTS software, the ability of the software for modeling piled raft foundations had been verified; Results show acceptable agreement between software output and monitored values and also outputs from other methods

¹ 3D Finite Element Analysis

³ Solid Element Method

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² Embedded Pile Method

1. Introduction

and software.

For a long time, designers used to consider two separate options for foundation design; shallow foundation consisting of raft foundation and deep foundations. However, in recent years they have found out that combining these two systems and simultaneously using the capacity of pile group and raft (in contact with the soil), would lead to economical design without losing efficiency and safety. These foundations are called raft foundations reinforced by piles or piled raft foundations. Bearing capacity of piled raft foundations is influenced by a complex interaction between soil below the structure and piled raft elements. There are four interactions between different elements which are shown in Figure 1 [1]:

- Soil-Pile Interaction
- Pile-Pile Interaction (the distance between piles influences the behavior of pile group; whether piles reach failure in singular or group mode)
- Raft- Soil Interaction
- Pile-Raft Interaction (imposed load from the raft over the soil causes more confinement and, consequently, increases the bearing capacity of the piles)

Being aware of these interactions and the use of analytical methods is critical for the reliable design of piled raft foundations.

Different methods had been developed for the analysis of piled raft foundations, which can be briefly categorized as follow :

- Methods based on Simplified calculations
- Computer-based Approximate methods
- More Rigorous computer-based methods

Simplified methods consist of Davis and Poulos [2], Randolph[3], van impe& clerk [4]and Berland [5] methods. Each of these methods has simplifications in the modeling soil profile and raft bearing.

Computer-based approximate methods consist of the following groups:

- The method which is based on" strip on spring" in which raft is modeled as a set of strip foundations and piles as springs (for example, Poulos [6])
- Methods which are based on "plane on string" in which raft is considered as a flexural plane and piles as springs (for example Clancy& Randolph[7], Poulos [6])

More Rigorous computer-based methods are as follow:

• Simplified finite element analysis :

These models usually consider foundation system as plane strain (Desai[8])or considers it as an axis-symmetric system (Hooper [9]), and finite difference analysis methods assuming plane-

strain or axis-symmetric conditions in commercial programs like FLAC, are placed in this category. (for example Hewitt &Gue [10])

• 3D-finite element and 3D-finite difference analysis :

Used in commercial software like PLAXIS 3D, FLAC 3D, ABAQUS, MIDAS GTS, and PLAXIS 3D FOUNDATION

• Boundary elements methods:

(like Butterfiled and Banerji [11], Sinha [12])

• Combined methods:

Methods that combine boundary elements methods for piles and finite elements methods for rafts.(for example Hain and Lee [13], Ta and Small [14], Franke et al.[15])

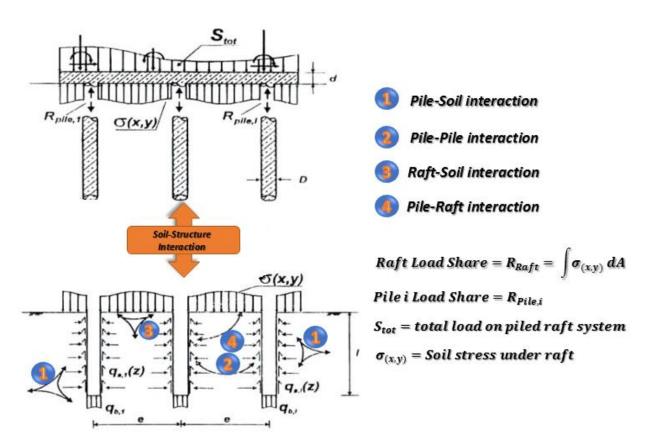


Fig. 1. Piled raft foundations which are consist of bearing elements of soil, raft and pile and interactions between these elements [1].

Simplified methods and computer-based approximate methods both have simplifications for considering the interaction between elements and soil behaviour modeling. So using these methods always have some errors. Therefore, these methods are used as an initial estimation in design and then use available 3D-FEM software for reliable results. 3D finite element methods are of the most reliable methods for the analysis of piled raft foundations which can consider

complex interaction between elements in these systems. Figure 2 shows a summary of the different methods of piled raft foundation analysis.



Fig. 2. Summary of the different methods of piled raft foundation analysis.

Sinha and Hanna (2016) performed a parametric study on piled raft foundations using ABAQUS software and the modified Drucker– Prager constitutive law. The research aim was to examine the effect of the governing parameters on the performance of piled raft foundations [16].

Deb and Kumar Pal (2019) used ABAQUS software package to study the response of a piled raft foundation under combined lateral and vertical loading and analyse the influence of vertical load on the lateral response of a piled raft foundation [17].

Mali and Singh (2018) simulated a large piled raft through 3-D finite element modelling with PLAXIS 3D. The objective of the present study was to investigate the effect of pile spacing, pile length, pile diameter and raft-soil stiffness ratio on the settlement, load-sharing, bending moments, and shear force behaviour of large piled-raft foundation [18].

Deb and Kumar Pal (2020) used 3D finite element modelling by ABAQUS FEM package to study the complex load sharing behaviour due to the presence of interaction effects. Based on this study they proposed a simplified model for the design of the piled raft foundation considering both the safety and serviceability conditions [19].

One of the main problems in applying 3D-FEM programs for analysis of piled raft foundations is that these models are very time consuming when the number of piles and elements increase, it leads to convergence problems for the numerical model. In this research, the capabilities of modern MIDAS GTS software for piled raft foundation analysis had been discussed. Different pile modeling techniques by various elements in this software have been discussed and compared. MIDAS GTS has many abilities for pile modeling and also has a wide range of elements which can analyze the piled raft foundation fast and accurately.

2. Research method

Since MIDAS GTS software is used in this research, at first software features are briefly discussed; it is a comprehensive program for finite element analysis with 2D and 3D modeling ability which is used for modeling of geotechnical operations like tunnels construction, foundations, excavations, leakage studies, Slope stability, Retaining structures, and consolidation and so on. It has an extensive library of rock and soil behaviour models (15 models) and also can perform various analyses.

In this software, pile modeling is available using SEM models, BSCM and EPM which significantly reduce the analysis time in comparison to solid elements and traditional pile modeling methods. In general finite elements software packages such as ABAQUS calculating axial forces and bending moments in each pile requires writing and implementing a python code. However, MIDAS GTS introduces Gauging Shell to estimate the moment, which is easier and axial forces and bending moments in each pile are calculated automatically. Only a few numbers of programs have such abilities. In summary, there are three methods for pile modeling in this software:

- SEM models for piles
- BSCM⁴ model
- EPM (or in other words line to solid interface model)

2.1. Models with solid elements

In Figure 3 the concepts of this model are shown, soil and piles are both modeled by solid elements. In these models, the connection between external nodes at the surface of the pile and soil is necessary (the interface).

⁴ Beam-Solid Connectivity Method

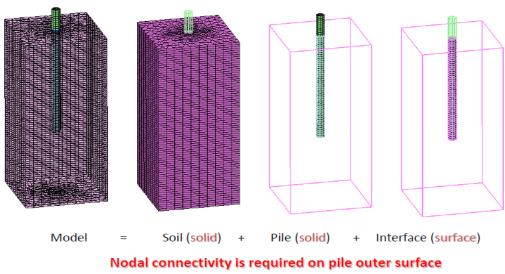


Fig. 3. Shcematic of a 3D model of pile with solid elements [20].

If it's needed to consider the displacement between pile and soil and a reduction in contact resistance between pile and soil in these models, surface interface elements as its shown in Figure 4 can be used for connecting solid elements.[21,22]

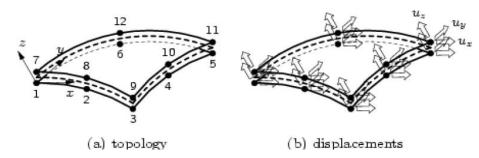


Fig. 4. Shcematic of Surface interface elements for solid to solid elements connection [23].

Some limitations of these models are as mentioned below:

- Defining the geometry of the model and meshing mechanism is complicated and timeconsuming.
- Many elements are created in these models which require a lot of computing time; especially in piled raft foundations with many piles resulting in a considerable computing time which is impractical for parametric studies.
- Axial Forces and bending moments in piles are not available directly for the user and should be calculated by the user which makes it difficult for parametric piled raft foundation analysis.

2.2. BSCM models

In these models, as its shown in Figure 5, soil is modelled as solid elements, but pile is modelled as a beam or a linear element, and if its needed to consider displacement between piles and soil or reduce the contact friction between them, line interface elements which is shown in Figure 6

can be applied; in these models nodal connectivity between pile and soil along the pile length is required.

Some of the disadvantages in beam-solid connectivity models are:[24]

- Nodal connectivity requirement makes the geometrical modeling and soil meshing processes difficult, although in MIDAS GTS it is done automatically by automatic meshing feature which only requires investigation about mesh quality.
- For piled raft foundations with a large number of piles, this modeling method leads to bigger models with more computing time, although computing time in these models is much lower than SEM models for piles.

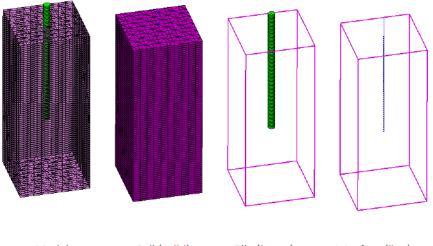




Fig. 5. Shcematic of 3D model of the pile with beam- solid connectivity elements[20].

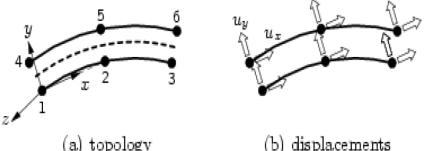
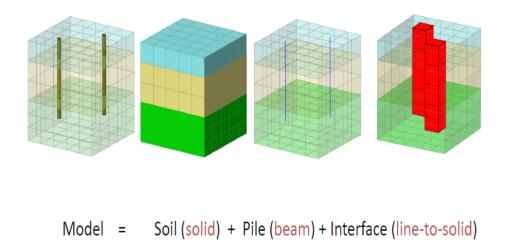


Fig. 6. Shcematic of Line interface elements for connecting beam element to solid element[23].

2.3. EPM models (line to solid interface model)

In these models, as it's shown in Figure 7, the soil is modelled by solid elements and the piles with a beam or line interface element. For modeling slippage between pile and soil and modeling friction resistance line to solid interface is used, which is shown in Figure 8, and for modeling tip bearing capacity, point to solid interfaces are used, as shown in Figure 9, which are applied by choosing "create pile element" option in the software. Creating these two elements is achieved by defining the min contact surfaces located between embedded pile element and soil element. The

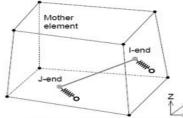
first type of a 'contact surface' that is used is a 'line to solid' interface that is used for modeling the friction between Pile and soil and lateral capacity and displacements at pile's sides. Another type of 'contact surfaces' that is used is 'point to solid interface' which is used for modeling the tip bearing capacity and displacement between soil and pile at the tip of the pile. In this way by defining connecting surface parameters and elements, it is possible to consider the displacement between pile and soil. In these models, nodal connectivity between beam and soil elements is not required, and soil meshing can be done separately from pile meshing which makes these models suitable for large piled raft foundations.



No Nodal connectivity required

Fig. 7. Shcematic of 3D model of piles with embedded piles or line to solid interface [20].

Line-to-solid interface elements for beam-to-solid connection:



Line-to-solid interface elements

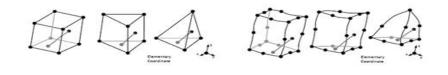
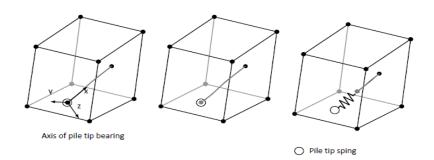


Fig. 8. Line interface elements for lateral friction [23].

In these models using point to solid interface elements gives us the ability for modeling tip bearing capacity. Figure 8 shows these elements; it is like a spring that connects the soil to the tip of the pile.



Model = Soil (solid) + Pile (beam) + Interface (point-to-solid)

The pile tip can be arbitrarily placed in the solid element.

It acts as an orientated spring connecting the soil to the pile tip.

Fig. 9. Point to solid connection interface elements for topside capacity [23].

Features of modeling by embedded pile element:

- Geometrical definition and pile-soil meshing can be done separately and independently.
- Crossing and intersection between line interfaces (beam connection elements) and soil elements can be calculated automatically.
- It is possible to model slippage with nonlinear friction-slip properties for line interface elements.
- Mesh refinement for the soil in these models is minimum which eventually decreases the calculations.

The summary of the comparison between three modeling methods for lateral skin friction and tip bearing capacity of piles are presented in tables 1&2, respectively.

Table1

Comparison between three kinds of models for side friction modeling.

Model Type	SEM	BSCM	Embedded piles or Line-to- solid interface model
Interface type	Surface	Line	Line - to - Solid
Nodal connectivity	Required	Required	Not required
Shear law	Coulomb friction plasticity	Defining the relation between friction and slippage per length	Defining the relation between friction and slippage per length
Friction stress – settlement displacement.	local	Averaged over circumference	Averaged over circumference
Transversal behaviour	Gap opening Possible	Rigid(high elastic stiffness)	Rigid(high elastic stiffness)
Variation over pile circumference	Considered	Not considered	Not considered

Comparison between	three kinds of models	for pile tip bearing capacity.	
Model Type	SEM	BSCM	Embedded piles or Line-to- solid interface model
Interface type	Surface	Point spring	Point - to - solid
Nodal connectivity	Required	Required	Required
Tip failure	(High refinement required)	Is considered by defining a relation between tip reaction and settlement	Is considered by defining a relation between tip reaction and settlement
Bearing stress - settlement displacement.	local	Averaged over tip surface	Averaged over tip surface
lateral behaviour	Coulomb friction Over pile section	Slipping	Slipping
Variation over the surface of the pile's tip	Considered	Not considered	Not considered

Table 2

Comparison between three kinds of models for pile tip bearing capacity.

The main difference between (SEM) and (EPM) is the fact that in EPM, side friction resistance, tip bearing capacity, and slip parameters are part of the inputs for the model, and they are defined as point-to-solid and line-to-solid for connecting surfaces; whereas it is not true in SEM, so before applying EPM for piled raft foundations, the model parameters should be calibrated. If the results of a pile test in a specific soil are available, calibration of these parameters will facilitate the modeling of large piled raft foundations and the required time for these processes and analysis also would be reduced, significantly, which is much lower in SEM. Additionally, pile's axial forces, bending moments and displacements between pile and soil and other required information are automatically determined by the software in post-processing section, and no user calculation is needed in contrast to SEM.

3. Results analysis and discussion

Figure 10 shows the results of pile loading test in Germany which is performed on a fixed pile in pre-reinforced clay in Frankfurt [15].

The groundwater is located 3.5m below the ground's level; piles have a diameter of 1.5 m and length of 9.5 m and placed in a consolidated clay layer. Loading system consists of 2 hydraulic jack that generates force over a reaction beam. This beam is supported by 16 anchors; they are placed vertically at a depth of 16-20 meters and a distance of 4 meters from piles under loading which minimizes the interaction between the pile and the system.[25]

Loading is done stepwise, and the amount of load at each step is fixed until the settlement rate is small. Applied load and related displacement are measured at the top of the pile. Also, the soil settlement is measured at different depths near the piles. In addition, loading cells which are mounted at the top of the piles, are capable of direct measuring of the forces. The calculated total load-settlement curve and the related fragmentation for side and tip resistance are shown in Figure 11.

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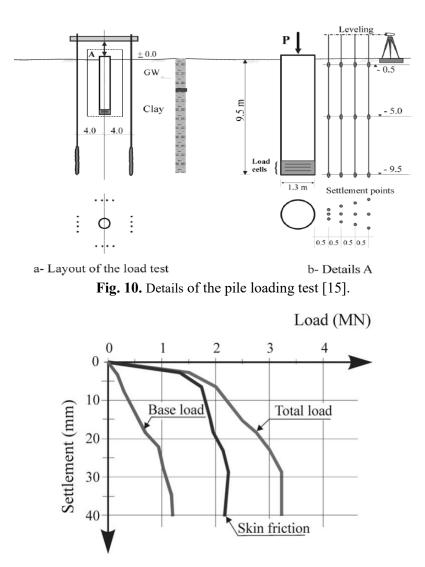


Fig. 11. Load-settlement curves for pile loading test [15].

Table 3.represents the material properties and constitutive model used in 3D-finite element models.

Table 3

Features of the behavioral model and materials in the pile's 3d loading test.

Constitutive model	Mohr coulomb Drained	
Type of behavior		
Unsaturated soil density	γ_{unsat}	20 kN/m ³
Saturated soil density	γ_{sat}	20 kN/m ³
Soil Elastic modulus	Е	6×10 ⁴ kN/m ²
Poisson Ratio	ν	0.3
Cohesion	С	20 kN/m ²
Friction Angle	Φ	22.5
Soil dilatancy angle	Ψ	0
At rest lateral pressure coefficient	\mathbf{k}_0	0.6

Figure 12 represent the model dimensions and number of elements and nodes for FEM modelling of pile using SEM and EPM methods. The mesh refinement was chosen as medium for both models.

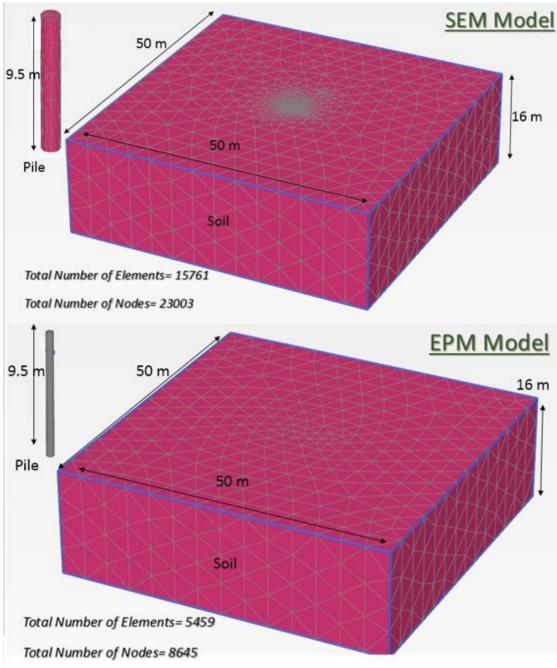


Fig. 12. Model properties for SEM and EPM models.

Figure 13 represents the comparison between total load-settlement curves indifferent pile modeling techniques and pile's loading test measurements.

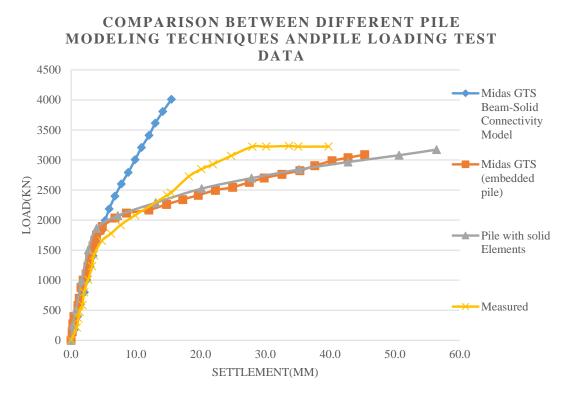


Fig. 13. A comparison between the pile's modeling techniques and test results.

As it can be seen, all models showed acceptable consistency within the linear part of the loadsettlement curve. When nonlinear behaviour of other models started, beam model (beam connected to solid elements) shows more rigid behaviour. The reason behind this is the fact that surface interface elements aren't used for side resistance reduction and slippage modeling between pile and soil which finally leads to more rigidity in the load-settlement curve of this model. However, in the following section through examples which are used for validation of piled raft foundation modeling, it can be observed that the results of the beam-connectivity model shows acceptable consistency with the results obtained by other models which are built by prominent researchers.

As it's mentioned earlier about comparing different pile modeling approaches, in EPM techniques, the side resistance and tip bearing capacity are not considered as part of the analysis results and instead applied as model's inputs. Also before applying the min piled raft foundation models, these two parameters and other parameters for the pile-soil interface model should be calibrated via pile loading test results. When calibration is done modeling piled rafts with a large number of piles can be done very easily and quickly and unlike pile modeling with solid elements, the values of forces, moments and many other parameters are available at post-processing section of MIDAS GTS software and does not require any calculation from users.

Figure 14 shows the results of fragmentation of load-settlement curves (pile skin resistance and base resistance) which are used for calibration and validation of EPM models in MIDAS GTS software.

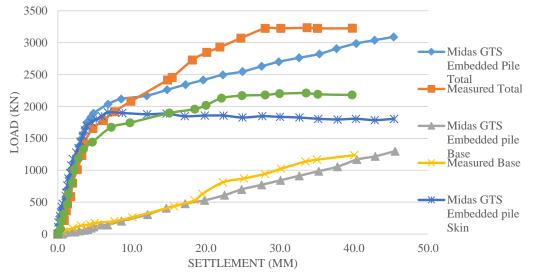


Fig. 14. Calibration and validation of skin resistance and base resistance for embedded piles model using the load-settlement curve of the pile loading test.

As it can be seen, there is a very good agreement between modeling results in embedded piles and pile loading test measurements.

3.1. 3D Modeling and validation of a piled raft foundation in MIDAS-GTS

3.1.1. First example

This example had been analysed by different authors like Poulos & Davis [1], Randolph [3], Sinha [12] (a combination of boundary and finite elements) and Ta& Small [14] (finite elements) via different techniques and different software like Plaxis3D, GARP, and GASP. Figure 15 shows the model's definition and geometry.

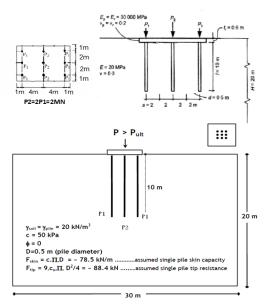


Fig. 15. Definition of hypothetical example by Poulos [1] which is used for software validation.

The average settlement value calculated by MIDAS GTS is 30 mm in this research. Figure 16 shows the comparison of MIDAS GTS settlement value with the value of settlement calculated by other methods and researches.

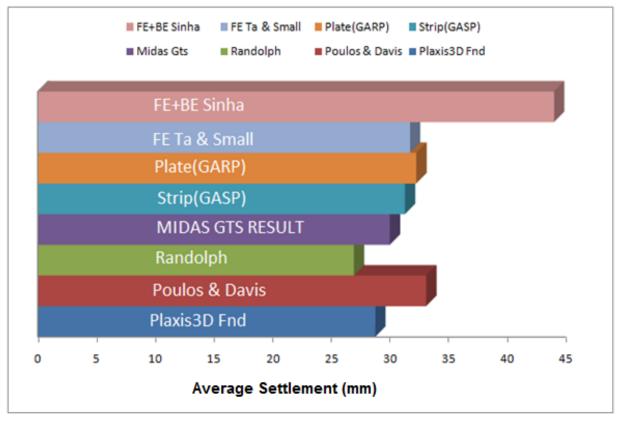


Fig. 16. Comparison of the results calculated using methods with calculated average settlement in MIDAS GTS for Poulos's hypothetical problem.

3.1.2. Second example

The second example is the validation process for Torhaus der Messe tower building; this building is constructed on a piled raft foundation with a dimension of 17.5 *24.5 m, each raft carries the 200MN load. Overly, in this piled raft foundation, there are 84 piles with diameter and length of 0.9 and 20 meters, respectively [26]. Figure 17 shows the model's definition, geometry and results. The maximum measured settlement is 140 mm, and the maximum settlement which is calculated in MIDAS GTS software is 166mm which indicates acceptable conformation with the measured settlement of the building.

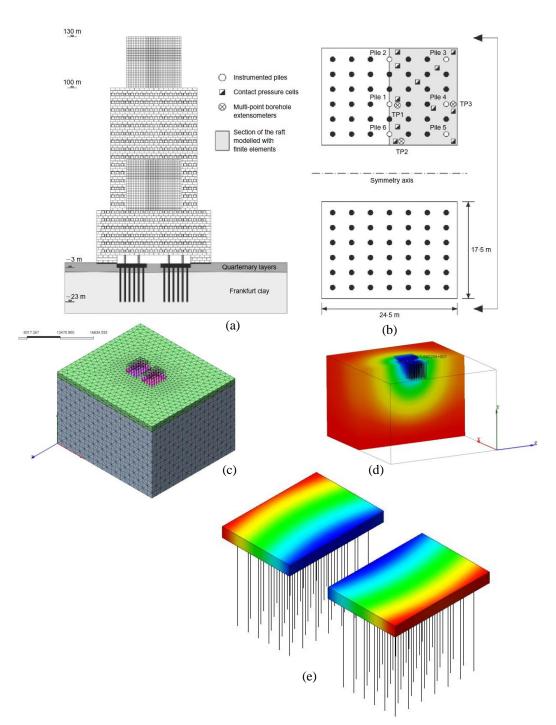


Fig. 17. (a), (b)Geometry details for Torhaus tower[26] (c), (d) and (e) validation process and result for Torhaus tower in MIDAS GTS software.

4. Summary and conclusions

Implementation of modern pile modeling methods such as EPM or BSCM would lead to a significant reduction in geometry complexity and calculation time in comparison to the conventional pile modeling techniques which are based on the use of SEM in the modeling

process. In these methods, pile forces and moments are calculated in the software, and there would be no need for manual calculations, unlike SEM-based models. Three pile modeling techniques which consist of solid element, BSCM and EPM are discussed and compared in section 3, and the following results have obtained:

- The main difference between SEM and the two other methods is the considerably more considerable amount of calculations required in SEM; conversely, in SEM, pile forces and moments are not calculated in the software and should be calculated manually (which is extremely tedious for a large number of piles).
- In EPM, parameters related to load-bearing capacity at skin, base, and slippage are considered as model's inputs. As a result, EPM is different in this issue with other methods. Prior inputting these parameters for piled raft foundation modeling, they should be calibrated; If the results of a pile loading test are available, this calibration will lead to the easy application of piled raft modeling techniques and the required time for computations and analysis would be much lower in comparison with SEM.
- For comparing the results of three pile modeling methods, they are compared with the actual data of a pile load-settlement test which showed an acceptable agreement in the linear area of loading- settlement curve.

In this research the MIDAS GTS software is verified for piled raft foundation modeling; results show acceptable conformation for software outputs and monitored values. MIDAS GTS has calculated the settlement with less than 18 percent of error. The software is also verified with Poulos hypothetical example which has been investigated by many researchers and methods; the amount of Settlement was an inacceptable agreement with settlement calculated by other methods and software; the settlement was virtually equal to the values computed by Plaxis3D foundation software.

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