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Improving the Behavior of Non-Uniform Tall Structures in Determining the Optimum Location of Belt Truss System

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ABSTRACT

The sensitivity of tall buildings subjected to the lateral loads is more than that of the gravity loads. Therefore, the conventional methods are not efficient yet, and new methods are proposed by designers to reduce the structural roof displacement, shear lag, overturning moment, and also increase the lateral resistance of the structures. In the design of tall structures, it is desirable to minimize the lateral stiffness of the structures for economic reasons. In this paper, the structure is modeled using the energy method and the continuous beam model. The outrigger's optimum position is calculated considering different loading patterns. It is assumed that the lateral stiffness of the structure changes with the height. An equivalent rotational spring is utilized to model the belt truss and outrigger system. The results show that the outrigger's optimum position depends on the type of the lateral load as well as how the stiffness changes in the height of the structure.

1. Introduction

Several resistant systems have been introduced to increase the efficiency of the framed tube structures. One of these systems is the combined system [1–9]. In this method, a relatively rigid

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shear core is combined with the framed tube and the outrigger systems. This leads to having a further reduction in the story displacement of the structure [10–25].

It is common to model the structure with a cantilever beam [7,22,26–30]. Besides, a linear rotational spring is utilized to model the interaction between the structure and the outrigger system. The linear rotational spring produces a moment that acts in the opposite direction of the structure subjected to the lateral load. Therefore, it reduces the lateral displacement of the structure. The important question is what position should be considered for the outrigger system to perform a better performance subjected to the lateral load. Minimizing the story displacement and drift ratio has been reviewed by several researchers to obtain the outrigger system's optimum position. Using this criterion, several researchers have calculated the outrigger's optimum location subjected to the dynamic and static loads (e.g., earthquake and wind loads) [1–3,8,10]. In this research, three and two-dimensional buildings are considered to study the effect of the outrigger's optimum position in the dynamic responses of the structures.

It is essential to determine the optimal values for engineering issues to reduce the cost and increase productivity. Besides, numerous methods have been used to determine optimal values in engineering problems [25,31–43]. The outrigger systems' behavior has been investigated by different literature [13,14,20,24,44–46]. Stafford Smith and Coull [45], as well as Taranath [14] proposed an approximate method for analyzing the belt truss systems. Also, Rahgozar *et al.* [24,44] examined the stress distribution and displacement of the combined system in tall buildings [24,44]. Hoenderkamp [4] and Hoenderkamp and Bakker [5] studied the effect of the shear wall system combined with the outrigger system on the lateral displacement of the structure. Kamgar *et al.* [10,19] examined the responses of a combined system subjected to the critical earthquakes.

The primary assumption for some of the existing literature is that the structure has a constant stiffness along with the height of the structure. In this paper, the main aim is to find the outrigger's optimum position using energy criteria for the non-uniform tall building subjected to the lateral load. Therefore, the behavior of the tall structure is modeled using a continuous beam method. Also, a linear rotational spring is used to simulate the effect of the outrigger system. This spring has been located at the position of this system. Then, the amount of energy stored in the outrigger system is calculated based on the location of the system along with the height of the structure for different types of lateral loading (i.e., concentrated, uniformly and triangularly distributed lateral loading). The maximization of the values of stored strain energy in the belt truss system results in finding the outrigger's optimum position.

2. Formulation

2.1. Calculation the strain energy stored in the equivalent rotational spring

The following assumptions are used to model the studied structure, as shown in Fig. (1):

- 1- The slabs are used to cover the floors of the stories, and it is assumed that they are rigid on their plane.
- 2- An equivalent linear elastic rotational spring is utilized to simulate the effect of the outrigger system.

- 3- A constant distance is considered between the adjacent columns through height of the structure.
- 4- The fixed-end conditions are considered for the shear core and framed tube at the bottom of the structure.
- 5- The materials are linear and homogeneous, and they follow Hooke's law.
- 6- The structure is symmetric around both of the main axes for all stories. Also, the coordinates for the center of mass and stiffness are the same, and therefore, the structure does not twist.
- 7- The connection of the outrigger to the shear core is assumed to be rigid. The pinned-end conditions are considered for the exterior columns. Therefore, only the axial forces are transmitted to the outer columns.

According to the above assumptions, the structure has variable lateral stiffness along with the height of the structure (see Fig. 1).

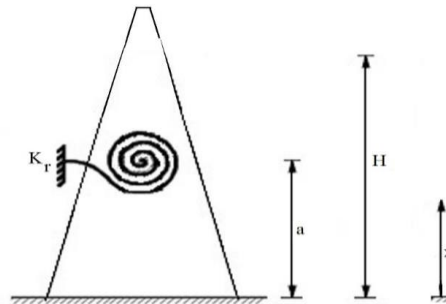


Fig. 1. A continuous model for the studied structure.

It should be noted that in Fig. (1), H , a , x , and Kr show the total height of the structure, the outrigger's position from the base of the structure, the distance from the bottom of the structure, and the linear rotational spring, respectively.

The amount of stored strain energy is calculated as follows:

$$E = \frac{1}{2} k_r \theta_a^2 \quad (1)$$

where θ_a shows the rotation of the outrigger system at its position.

The external work is defined as the done work subjected to lateral loading (e.g., wind and earthquake load). The members of the structure store this work as strain energy. Therefore, the outrigger and belt truss system can store a part of this input energy as a particular member of the structure. Finally, when the energy absorbed by the spring is at its maximum value, the spring has located at its best position. This implies that other structural members undergo less input energy subjected to the lateral load, and therefore it reduces the damage in the structural members.

Therefore, the derivative of the strain energy should be calculated relative to the position of the rotational spring from the bottom of the structure (a). This value is set to zero to find the outrigger's optimum location (i.e., $dE/da = 0$).

In the next section, the values of θ_a and k_r are computed as a function of the position of the rotational spring (a). Also, the following section presents the values of equivalent stiffness for the outrigger system.

2-2. Calculation the equivalent stiffness of the rotational spring

In this section, it is assumed that the lateral (bending stiffness $EI(x)$) and axial stiffness ($AE(x)$) of the structure change along with the height of the structure. These values are considered as Eq. (2) based on Ref. [18]:

$$EI(x) = EI_0(1 + \beta x)^{m+2}$$

$$AE(x) = AE_0(1 + \beta x)^{m'+1}$$
(2)

where x , EI_0 , and AE_0 present the height of the structure from the base, the values of bending and axial stiffness at the bottom of the structure, respectively. The parameters m, m' , and β are selected in such a way that they can trace the flexural and axial stiffness along with the height of the building.

In the combined system, the structure's curvature will not be like a single beam when the building is exposed to the horizontal loads. The belt truss and outrigger system connect the shear core to the external columns. This leads to the formation of a turning point. Therefore, according to Fig. (2), the set of external columns and the outrigger system reduce the lateral deformations and base moment.

It has been known that the relationship between the moment (M) and the rotation of the rotational spring can be written as follows, according to Figs. (1-3):

$$M = K_r \theta_a$$
(3)

θ_a shows the amount of rotation of the cantilever beam at the outrigger's position. Also, the following equation can be obtained based on Fig. (2):

$$M = F \times d$$
(4)

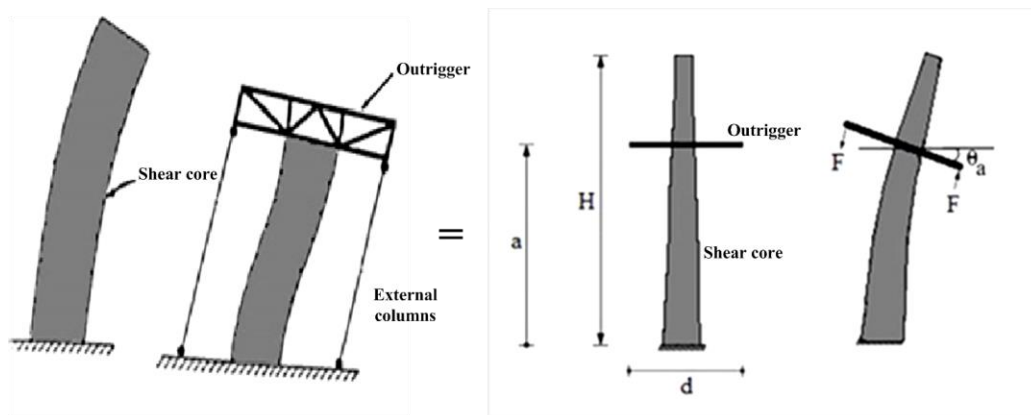


Fig. 2. Reduction in the values of structural displacement considering the effect of the outrigger system.

The amount of axial elongation for the exterior columns (δ) can be computed by considering Eq. (2) subjected to the axial force F (see Fig. 2).

$$\delta = \int_0^a \frac{F}{AE(x)} dx = \frac{F}{\beta AE_0 m'} \left(1 - \frac{1}{(1+\beta a)^{m'}} \right) \quad (5)$$

Also, the produced moment in the rotational spring and the displacement created in the exterior columns connected to the outrigger can be calculated using Eqs. (6 and 7).

$$M = F \times d \quad (6)$$

$$\delta = \theta_a \times \frac{d}{2} \quad (7)$$

On can computed the equivalent stiffness of the rotational spring using Eqs. (5-7).

$$K_r = \frac{\beta AE_0 m' d^2}{2 \times (1 - (1 + \beta a)^{-m'})} \quad (8)$$

As can be deduced from Eq. (8), the equivalent stiffness for the outrigger system depends on the axial stiffness of the exterior columns and also rate of changes in the axial stiffness of the surrounding columns through the height of the structure (AE_0 , m' , and β), the distance between the adjacent exterior columns (d) and the outrigger's position (a).

2-3. Calculation the outrigger's optimum position subjected to the uniformly distributed lateral load

Based on the superposition principle, the value of θ_a (rotation angle of beam at the outrigger's position) can be calculated by the summation of $\theta_{a,1}$ (due to lateral loading) and $\theta_{a,2}$ (created by the concentrated moment applied to the structure in the outrigger's location). According to Fig. (3), the $\theta_{a,1}$ can be calculated as follows when a uniformly distributed lateral load will be applied to the structure with intensity w :

$$\begin{aligned} \theta_{a,1} = \int_0^a \frac{M(x)}{EI(x)} dx = \int_0^a \frac{(wHx - \frac{wH^2}{2} - \frac{wx^2}{2})}{EI_0(1+\beta x)^{m+2}} dx = \frac{wH}{\beta^2 EI_0} \left(\frac{1}{(m+1)(1+\beta a)^{m+1}} - \frac{1}{m(1+\beta a)^m} \right. \\ \left. + \frac{1}{m(m+1)} \right) + \frac{wH^2}{2\beta EI_0} \left(\frac{1}{(m+1)(1+\beta a)^{m+1}} - \frac{1}{(m+1)} \right) - \frac{w}{2\beta^3 EI_0} \left(\frac{(1+\beta a)^{1-m}}{(1-m)} + \frac{2}{m(1+\beta a)^m} \right. \\ \left. - \frac{1}{(m+1)(1+\beta a)^{m+1}} - \frac{2}{m(1-m^2)} \right) \end{aligned} \quad (9)$$

Also, the value of $\theta_{a,2}$ parameter is equal to:

$$\theta_{a,2} = \int_0^a \frac{K_r \times \theta_a}{EI(x)} dx = - \frac{K_r \times \theta_a}{(m+1)\beta EI_0} \left(\frac{1}{(1+\beta a)^{m+1}} - 1 \right) \quad (10)$$

Therefore, the value of the θ_a parameter can be computed as follows:

$$\theta_a = \theta_{a,1} + \theta_{a,2} = \frac{I_1}{I_2}$$

$$I_1 = \frac{wH}{\beta^2 EI_0} \left(\frac{1}{(m+1)(1+\beta a)^{m+1}} - \frac{1}{m(1+\beta a)^m} + \frac{1}{m(m+1)} \right) + \frac{wH^2}{2\beta EI_0} \left(\frac{1}{(m+1)(1+\beta a)^{m+1}} - \frac{1}{(m+1)} \right) - \frac{w}{2\beta^3 EI_0} \left(\frac{(1+\beta a)^{1-m}}{(1-m)} + \frac{2}{m(1+\beta a)^m} - \frac{1}{(m+1)(1+\beta a)^{m+1}} - \frac{2}{m(1-m^2)} \right)$$

$$I_2 = 1 + \frac{AE_0 m' d^2}{2EI_0(m+1)} \frac{[(1+\beta a)^{-(m+1)} - 1]}{[1 - (1+\beta a)^{-m}]}$$

Now using Eqs. (1), (8) and (11), the outrigger's optimum position can be calculated by setting the derivative of the strain energy stored in the spring to zero. The derivative should be computed relative to the position of the rotational spring (a). By definition a new variable as $\xi = (1 + \beta a)$, for this state, the outrigger's optimum location can be computed as follows:

$$\frac{dE}{da} = \frac{dE}{dK_r} \frac{dK_r}{da} + \frac{dE}{d\theta_a} \frac{d\theta_a}{da} = 0 \Rightarrow \left\{ \frac{H}{\beta} \left(\frac{1}{(m+1)\xi^{m+1}} - \frac{1}{m\xi^m} + \frac{1}{m(m+1)} \right) + \frac{H^2}{2} \left(\frac{1}{(m+1)\xi^{m+1}} - \frac{1}{(m+1)} \right) - \frac{1}{2\beta^2} \left(\frac{1}{(1-m)\xi^{m-1}} + \frac{2}{m\xi^m} - \frac{1}{(m+1)\xi^{m+1}} - \frac{2}{m(1-m^2)} \right) \right\} \times \left\{ \frac{-m'\xi^{-(m'+1)}}{2(1-\xi^{-m'})} \right\} + \left\{ \frac{H}{\beta} (-\xi^{-(m+2)} + \xi^{-(m+1)}) - \frac{H^2}{2} \xi^{-(m+2)} - \frac{1}{2\beta^2} (\xi^{-m} - 2\xi^{-(m+1)} + \xi^{-(m+2)}) \right\} - \left\{ \frac{I_3}{I_4} \right\} \times \left\{ \frac{I_5}{I_6} \right\} = 0$$

where

$$I_3 = \frac{H}{\beta^2} \left(\frac{1}{(m+1)\xi^{m+1}} - \frac{1}{m\xi^m} + \frac{1}{m(m+1)} \right) + \frac{H^2}{2\beta} \left(\frac{1}{(m+1)\xi^{m+1}} - \frac{1}{(m+1)} \right) - \frac{1}{2\beta^3} \left(\frac{\xi^{1-m}}{(1-m)} + \frac{2}{m\xi^m} - \frac{1}{(m+1)\xi^{m+1}} - \frac{2}{m(1-m^2)} \right)$$

$$I_4 = 1 + \frac{AE_0 m' d^2}{2(m+1)EI_0(1-\xi^{-m'})} \left(\frac{1}{\xi^{m+1}} - 1 \right)$$

$$I_5 = AE_0 m' d^2 \times \left\{ -(m+1)\beta \xi^{-(m+2)} (1-\xi^{-m'}) + m'\beta \xi^{-(m'+1)} (1-\xi^{-(m+1)}) \right\}$$

$$I_6 = 2(m+1)EI_0(1-\xi^{-m'})^2$$

By considering a constant stiffness along with the height of the structure ($m = -2$ and $m' = -1$), the outrigger's optimum location is computed as $a = 0.4417H$ using Eq. (12).

2-4. Calculation the outrigger's optimum position subjected to the concentrated lateral load with intensity P

Similar to the previous section, when the structure is loaded by a concentrated lateral load (P) located at the top of the structure, the $\theta_{a,1}$ is obtained as follows:

$$\theta_{a,1} = \int_0^a \frac{M(x)}{EI(x)} dx = \int_0^a \frac{P(x-H)}{EI_0(1+\beta x)^{m+2}} dx = \frac{P}{\beta^2 EI_0} \left\{ \frac{1}{(m+1)(1+\beta a)^{m+1}} - \frac{1}{(m+1)} - \frac{1}{m(1+\beta a)^m} + \frac{1}{m} \right\} + \frac{PH}{\beta EI_0} \left\{ \frac{1}{(m+1)(1+\beta a)^{m+1}} - \frac{1}{(m+1)} \right\} \quad (14)$$

Also, the value of the $\theta_{a,2}$ parameter is similar to Eq. (10). Therefore, the value of the angle θ_a can be computed as follows:

$$\theta_a = \theta_{a,1} + \theta_{a,2} = \frac{I_7}{I_8}$$

$$I_7 = \frac{P}{\beta^2 EI_0} \left\{ \frac{1}{(m+1)(1+\beta a)^{m+1}} - \frac{1}{(m+1)} - \frac{1}{m(1+\beta a)^m} + \frac{1}{m} \right\} + \frac{P}{\beta EI_0} \left\{ \frac{1}{(m+1)(1+\beta a)^{m+1}} - \frac{1}{(m+1)} \right\} \quad (15)$$

$$I_8 = 1 - \frac{AE_0 m' d^2}{2EI_0(m+1)(1-(1+\beta a)^{-m'})} \left(1 - \frac{1}{(1+\beta a)^{m+1}} \right)$$

Now using Eqs. (1), (8) and (15), the outrigger's optimum position can be calculated by setting the derivative of the strain energy stored in the spring to zero. Therefore, the outrigger's optimum position can be computed as follows:

$$\frac{dE}{da} = \frac{dE}{dK_r} \frac{dK_r}{da} + \frac{dE}{d\theta_a} \frac{d\theta_a}{da} = 0 \Rightarrow \left\{ \frac{1}{\beta^2} \left(\frac{1}{(m+1)\xi^{m+1}} - \frac{1}{m\xi^m} + \frac{1}{m} - \frac{1}{m+1} \right) + \frac{1}{\beta} \left(\frac{1}{(m+1)\xi^{m+1}} - \frac{1}{(m+1)} \right) \right\} \times \left\{ \frac{-\beta m' \xi^{-(m'+1)}}{(1-\xi^{-m'})} \right\} + 2 \times \left\{ -\frac{\beta+1}{\beta} \xi^{-(m+2)} + \frac{1}{\beta} \xi^{-(m+1)} + \frac{I_9}{I_{10}} \right\} = 0 \quad (16)$$

where

$$I_9 = \left\{ \frac{1}{\beta^2} \left(\frac{1}{(m+1)\xi^{m+1}} - \frac{1}{m+1} - \frac{1}{m\xi^m} + \frac{1}{m} \right) + \frac{1}{\beta} \left(\frac{1}{(m+1)\xi^{m+1}} - \frac{1}{(m+1)} \right) \right\} \times \left\{ \frac{AE_0 m' d^2}{2EI_0(m+1)} \right\} \times \left\{ (m+1)\beta \xi^{-(m+2)} (1-\xi^{-m'}) - m'\beta \xi^{-(m'+1)} (1-\xi^{-(m+1)}) \right\} \quad (17)$$

$$I_{10} = \left\{ 1 - \frac{AE_0 m' d^2}{2(m+1)EI_0} \left(\frac{1-\xi^{-(m+1)}}{1-\xi^{-m'}} \right) \right\} \left\{ 1 - \xi^{-m'} \right\}^2$$

Also, assuming a constant stiffness along with the height of the structure ($m = -2$ and $m' = -1$), for this state, the outrigger's optimum position is calculated to be equal to $a = 0.6667H$.

2.5. Calculation the outrigger’s optimum position subjected to the triangularly distributed lateral load with intensity W

Like the previous section, when the structure is loaded by a triangularly distributed lateral load with intensity w , the $\theta_{a,1}$ is calculated as follows:

$$\begin{aligned} \theta_{a,1} = \int_0^a \frac{M(x)}{EI(x)} dx = \int_0^a \left(-\frac{wH^2}{3} + \frac{wHx}{2} - \frac{wx^3}{6H} \right) \frac{1}{EI_0(1+\beta x)^{m+2}} dx = \frac{-w}{6\beta^4 EI_0 H} \left\{ -\frac{1}{(m-2)(1+\beta a)^{m-2}} + \frac{1}{(m-2)} \right. \\ \left. + \frac{3}{(m-1)(1+\beta a)^{m-1}} - \frac{3}{m-1} - \frac{3}{m(1+\beta a)^m} + \frac{3}{m} + \frac{1}{(m+1)(1+\beta a)^{m+1}} - \frac{1}{(m+1)} \right\} \\ + \frac{WH}{2\beta^2 EI_0} \left\{ -\frac{1}{m(1+\beta a)^m} + \frac{1}{m} + \frac{1}{(m+1)(1+\beta a)^{m+1}} - \frac{1}{(m+1)} \right\} \\ - \frac{WH^2}{3\beta EI_0(m+1)} \left\{ 1 - \frac{1}{(1+\beta a)^{m+1}} \right\} \end{aligned} \tag{18}$$

Also, the value of the angle $\theta_{a,2}$ is similar to Eq. (10). Therefore, the value of the angle θ_a can be computed as follows:

$$\begin{aligned} \theta_a = \theta_{a,1} + \theta_{a,2} = \frac{I_{11}}{I_{12}} \\ I_{11} = \frac{-w}{6\beta^4 EI_0 H} \left\{ -\frac{1}{(m-2)(1+\beta a)^{m-2}} + \frac{1}{(m-2)} + \frac{3}{(m-1)(1+\beta a)^{m-1}} - \frac{3}{m-1} - \frac{3}{m(1+\beta a)^m} \right. \\ \left. + \frac{3}{m} + \frac{1}{(m+1)(1+\beta a)^{m+1}} - \frac{1}{(m+1)} \right\} + \frac{WH}{2\beta^2 EI_0} \left\{ -\frac{1}{m(1+\beta a)^m} + \frac{1}{m} \right. \\ \left. + \frac{1}{(m+1)(1+\beta a)^{m+1}} - \frac{1}{(m+1)} \right\} - \frac{WH^2}{3\beta EI_0(m+1)} \left\{ 1 - \frac{1}{(1+\beta a)^{m+1}} \right\} \\ I_{12} = 1 + \frac{AE_0 m' d^2}{2EI_0(m+1)} \frac{[(1+\beta a)^{-(m+1)} - 1]}{[1 - (1+\beta a)^{-m}]} \end{aligned} \tag{19}$$

Now using Eqs. (1), (8) and (19), the outrigger’s optimum position can be calculated as follows:

$$\begin{aligned} \frac{dE}{da} = \frac{dE}{dK_r} \frac{dK_r}{da} + \frac{dE}{d\theta_a} \frac{d\theta_a}{da} = 0 \Rightarrow \left\{ -\frac{1}{6H\beta^4} \left(-\frac{1}{(m-2)\xi^{m-2}} + \frac{1}{(m-2)} + \frac{3}{(m-1)\xi^{m-1}} \right. \right. \\ \left. - \frac{3}{(m-1)} - \frac{3}{m\xi^m} + \frac{3}{m} + \frac{1}{(m+1)\xi^{m+1}} - \frac{1}{m+1} \right) + \frac{H}{2\beta^2} \left(-\frac{1}{m\xi^m} + \frac{1}{m} + \frac{1}{(m+1)\xi^{m+1}} - \frac{1}{(m+1)} \right) \\ \left. - \frac{H^2}{3\beta} \left(-\frac{1}{(m+1)\xi^{m+1}} + \frac{1}{(m+1)} \right) \right\} \times \left\{ \frac{-\beta m' \xi^{-(m'+1)}}{2(1-\xi^{-m'})} \right\} - \frac{1}{6H\beta^3} \times (\xi^{-(m-1)} - 3\xi^{-m} + 3\xi^{-(m+1)} - \xi^{-(m+2)}) \\ + \frac{H}{2\beta} \times (\xi^{-(m+1)} - \xi^{-(m+2)}) - \frac{H^2}{3} \xi^{-(m+2)} + \left\{ \frac{I_{13}}{I_{14}} \right\} \times \left\{ -\frac{1}{6H\beta^4} \left(-\frac{1}{(m-2)\xi^{m-2}} + \frac{1}{(m-2)} + \frac{3}{(m-1)\xi^{m-1}} \right. \right. \\ \left. - \frac{3}{(m-1)} - \frac{3}{m\xi^m} + \frac{3}{m} + \frac{1}{(m+1)\xi^{m+1}} - \frac{1}{m+1} \right) + \frac{H}{2\beta^2} \left(-\frac{1}{m\xi^m} + \frac{1}{m} + \frac{1}{(m+1)\xi^{m+1}} - \frac{1}{(m+1)} \right) \\ \left. - \frac{H^2}{3\beta} \left(-\frac{1}{(m+1)\xi^{m+1}} + \frac{1}{(m+1)} \right) \right\} \end{aligned} \tag{20}$$

where

$$\begin{aligned}
 I_{13} &= \frac{AE_0 m' d^2 I_{15}}{2(m+1)EI_0 [1-\zeta^{-m'}]^2} \\
 I_{14} &= I + \frac{AE_0 m' d^2}{2(m+1)EI_0} \left(\frac{\zeta^{-(m+1)} - 1}{1-\zeta^{-m'}} \right) \\
 I_{15} &= (m+1)\beta \zeta^{-(m+2)} [1-\zeta^{-m'}] + m'\beta [\zeta^{-(m+1)} - 1] \zeta^{-(m'+1)}
 \end{aligned} \tag{21}$$

Therefore, by assuming a constant stiffness along with the height of the structure ($m = -2$ and $m' = -1$), for this state, the outrigger's optimum position can be computed as $a=0.4903H$ using Eq. (21).

3. Results

Table (1) shows a comparison between the results of the present study and other existing research.

Table 1.

A comparison between the outrigger's optimum position obtained by different methods.

Load type	Outrigger's optimum position			
	Present study	Kangar and Rahgozar [10] and Jahanshahi and Rahgozar [6]	SAP 2000 [6]	Rahgozar and Sharifi [24]
Uniformly distributed lateral load	0.4417 H	0.441 H	0.400 H	0.545 H
Concentrated load	0.6667 H	0.667 H	0.700 H	0.667 H
Triangularly distributed lateral load	0.4903 H	0.490 H	0.530 H	0.571 H

It can be concluded from Table 1 that a good agreement exists between the results of the present study and existing research. Besides, the results show that the outrigger's optimum position depends on the type of the lateral load and also how the axial and bending stiffnesses of the structure change through the height of the structure (see Eqs. (12, 16, and 20) and Table 1). It should be noted that using the introduced equations (see Eqs. 12, 16, and 20), the outrigger's optimum location can be calculated for the desired stiffness changes along with the height of the structure.

4. Conclusion

Here, the outrigger's optimum position is determined for the desired stiffness changes along with the height of the structure. Therefore, the structure is modeled using the energy method and the

continuous beam model. Then, the outrigger's optimum position is determined for different lateral loading patterns. It is assumed that the lateral stiffness of the structure varies through the height of the structure. Besides, an equivalent rotational spring is selected to simulate the effect of the outrigger and belt truss system. Finally, the outrigger's optimum location can be calculated by setting the derivative of the strain energy stored in the spring to zero. The results show that the outrigger's optimum position depends on the type of lateral load and the structure's stiffness.

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