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Application of FOSM and Chebyshev's Theorem Concept to Predict Compressive Strength Attributes of Corncob Ash Embedded Cement Concrete

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ABSTRACT

In the present study the physical and mechanical properties of Corn Cob Ash (CCA) embedded cement concrete of mix proportions 1:1.6:2.6 and water-cement ratios of 0.45 were examined and compared with conventional cement concrete. A total of 96 concrete cubes of size 150 × 150 × 150 mm³ with different percentages by volume of CCA to ordinary Portland cement of grade 30Mpa in the order 0:100, 5:95, 10:90 and 15:85 were cast, tested and their physical and mechanical properties were determined. The specific gravity of the CCA was 3.15. The cubes were cured in a laboratory setup and compressive strength measures of 7, 14, 21, 28, 35, 42 and 56 days were observed. The compressive strength tests on cement concrete by replacing cement with 5% of CCA showed quite satisfactory results at 28days, 35days, 42days and 56days curing period compared to conventional concrete. But 15% CCA replacing for cement did not meet the satisfactory strength attributes. The probability analysis for compressive strength attributes were studied and presented. Chebyshev's theorem was utilised to find how spread the data is from mean. First order second moment method was used to found mean compressive strength and standard deviations. As the CCA replacement level increased, the compressive strength and workability decreased. However the concrete cubes have gained strength with age. The results indicate that CCA is an adoptable mineral admixture and pozzolan with the substitution level of 5% cement, with no unfavourable consequences for other different attributes of the hardened cement concrete.

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1. Introduction:

Concrete is one of the universally-used construction materials in the world today. The important reasons for increase in cement consumptions is necessity of better infrastructure facilities for humans, improvement in standard of living of them and burst of population. The primary component of cement is limestone. The cement production contributes to large amount of CO_2 emission by combustion of lime stone fuels at kiln [1] Hence, increment in concrete production will prompt increment of Greenhouse Gas emanations. Given its high smoke emissions and importance of human health, control in cement production is an obvious way to look to reduce greenhouse gas emissions [1,2] The generation of cement discharges greenhouse gas outflow both legitimately and by indirectly: the heating of limestone discharges CO_2 straightforwardly, while the burning of non-renewable fossils to heat the furnace (kiln) in another way brings about CO_2 emanations. The direct smoke emissions of cement production occur through a chemical process called calcination. Indirect smoke emissions are produced by burning fossil fuels to heat the kiln. Increment in cement concrete usage was intended for the cement generation has expanded causing an expansion in carbon emanations. This increase in carbon emissions has led to expanded efforts on studying alternative materials as a substitution to cement to mitigate greenhouse gas emanation.

Use of agricultural waste is considered as boon in replacement material of cement and one of the best solutions in minimizing water, land and air pollution [1–30]. These agricultural wastes are corn cob from maize, wheat husk from wheat straw, groundnut shell from groundnut and rice husk from paddy. Corn cob, wheat straws, also, plane leaf ashes could be advantageously used to get durable concrete [4]. Wheat straw cinders, and different ashes, have high residual silica. Examples, calcined sunflower ashes, and calcined tobacco ashes, both of which might be helpful as pozzolans in cement[22]. There had been different research endeavors on the utilization of CCA and another pozzolan as a substitution for cement. Suwanmaneechot et, al (2015) had presented a study on the effects of heat treatment on chemical composition, physical properties and engineering properties of CCA [5]. Raheem et, al (2010) had investigated the effects of admixtures on the properties of CCA cement concrete [18]. Raheem (2008) studied the workability and compressive strength characteristics of CCA blended cement concrete [20] . Raheem (2009) also studied the development of CCA Blended Cement [22]. Levitas et, al have studied the phase transitions effects in martensite nature materials, which occurs in Calcium and Aluminium at different temperature in Cementitious material [31–33]. These types of researches in the materials field are the future trend. CCA is obtained in the boilers of animal feed industry. CCA can be used as Pozzolanic constituent in concrete as it fulfils the requirement for such material by combining SiO2 and Al_2O_3 more than 70%. It present while investigating the strength gain characteristics of corn-cob ash concrete. The main aim of this study is to investigate the potential of CCA as partial replacement of cement in concrete and to investigate strength gaining attributes of concrete. The use of replacement material offers cost reduction, energy saving, fabrication of superior quality product and fewer hazards in environment.

Very limited information is available on the prediction of compressive strength attributes of Corncob ash embedded cement concrete. Indeed, at present, general research in the area of forecasting of hardened properties of Corncob ash embedded cement concrete is still in infancy. Although statistical methods are well established for assessing the results of concrete testing's, its effect of forecasting of results and its ranges of Corncob ash embedded cement concrete has not been extensively studied. Therefore, it is sensible to apply the First Order

Second Moment (FOSM) approach of probability concept and Chebyshev's theorem concept in order to realise the range and strength of Corncob ash embedded cement concrete. Casting and testing of cement concrete requires lot of skilled workers and repetitive laboratory work procedure. So there is a need to do investigation on forecasting methods of compressive strength attributes of Corncob ash embedded cement concrete and their probabilistic study to prognosticate its strength ranges. The FOSM approach of probability concept and Chebyshev's theorem concept will support to predict the range of a required CCA replaced cement concrete material for specific application.

2. Experimental Studies

Mix Proportioning and Casting of Concrete Cubes:

In the present study batching by volume of concrete ingredients was adopted as per the norms of IS-10262-2009 code [34]. A mix of 1: 1.6: 2.6 (cement: fines: coarse aggregates) was examined with water/cement ratios of 0.45. The fines utilized were M-sand. Granite was utilized as a coarse material. A steel mould of size 150 × 150 × 150 mm^3 was utilized for fabrication of concrete blocks. The mould was assembled before the wet mixing of concrete and appropriately greased up for simple expulsion of hardened blocks of concrete. Each block was prepared in percentage by volume of CCA to cement as a binder in the ratio of 0:100, 05:95, 10:90 and 15:85. The wet concrete mix was consistently mixed until it arrived at a plastic state after which it was set in the moulds and vibrated for 01 moment on vibrating table experiment setup to evacuate air bubbles assuming any. The moulded concrete cubes were given 24 hours to set before de-moulding. They were then immersed into large curing tank in order to increase the strength of the concrete, promote hydration, eliminate shrinkage and absorb heat of hydration until the age of test. Cubes were water cured for 7 days, 14 days, 21 days, 28 days, 35 days, 42 days and 56 days. The cubes (figure 1) were weighed before testing and the densities of cubes at different time of testing were computed. Before testing, the test blocks were brought out of the water tank and kept in the open air for around 02 hours before the test. The strength attributes were the average values of three specimens examined for each by utilizing mechanized compression testing machine. The workability, density, and compressive strength were inspected for all the mixes. Table 1 shows the mix proportioning details for various concrete mixes.

In the present study the probabilistic analysis and probable range of achieved experimental results were done by chebyshev's theorem concept (by utilizing the equation -1 and 2) and first order second moment (FOSM) method.

Table 1

Concrete mix proportions.

Mixes	CCA %	CCA in kg/m^3	Cement in kg/m^3	Water in liter/ meter	Water-cement ratio	Coarse aggregate in kg/m^3	Fine aggregate in kg/m^3
M1	00	00.0	426.0	192	0.45	1108.9	682.7
M2	05	21.3	404.7	192	0.45	1108.9	682.7
M3	10	42.6	383.4	192	0.45	1108.9	682.7
M4	15	63.9	362.1	192	0.45	1108.9	682.7

Fig. 1. Concrete cube for compressive strength testing.

Here, by utilizing the concept of Chebyshev's theorem, it is accounted that, there were 95% of data within the interval of 4.47 MPa from the mean value of compressive strength.
Let, k = number of standard deviations spread from the mean. Then chebyshev's theorem concept states that,

$(1-(1/k^2)) = 0.95$ (1)

Note: here '0.95' represents 95% probability of the predicting variable parameter.

$k^2 = (1 / (1-0.95))$

$k^2 = (1 / 0.05)$

$k = 4.47$

Range of compressive strength is calculated by using the expression – 1.

Range = $(\mu\text{-}k\sigma)$ - $(\mu+k\sigma)$ (2)

From FOSM Method the mean and standard deviation of compressive strength is determined as shown in equations 3 and 4. Mean compressive strength of concrete is the ratio of product of mean of breadth and depth over applied mean load. That is,
From the concept of FOSM approach,
Let, R = Compressive strength of corncob ash replaced cement concrete in N/mm^2,
here 'μR' and 'σR' represents mean and standard deviations of variable 'R' with unit N/mm^2.
P = Load in kN, μ_P = Mean Load in kN, σp = Standard deviation of load in kN
b = Breadth of specimen in mm, μb = Mean breadth in mm, σb = Standard deviation of breadth in mm
d = Depth of specimen in mm, μd = Mean depth in mm, σb = Standard deviation of depth in mm

$$\text{Mean compressive strength } (\mu R) = \frac{\text{Mean of Load}}{Mean\ of\ Breadth\ x\ Mean\ of\ Length} \quad (3)$$

Then standard deviations of compressive strength (σ_R^2) are calculated as follows

$$\sigma_R{}^2 = \left(\frac{1}{\mu b\ \mu d}\right)^2 \sigma_P{}^2 + \left(\frac{-\mu P}{\mu b^2 \mu d}\right)^2 \sigma_b{}^2 + \left(\frac{-\mu P}{\mu b\ \mu d^2}\right)^2 \sigma_d{}^2 \quad (4)$$

3. Results and Discussions:

The results of performed tests of varied CCA concrete are tabulated in tables 2 to 6. Table-2 shows the slump values of fresh concrete mixes. Table-3 shows the density of the concrete cubes. Table-4 provides the average compressive strengths of the concrete cubes examined. Table-5 depicts the range of dispersion of compressive strength of fabricated concrete cubes up to standard-deviation of 4.472 (which was decided by the application of concept of chebyshev's theorem for finding out 95% accurate stochastic value). Table - 6 depicts the mean compressive strengths of concrete cubes by the FOSM method.

Table 2
Workability of concrete.

Mix	CCA%	Slump in mm
M1	00	215.0
M2	05	15.2.5
M3	10	070.0
M4	15	058.0

Table 3
Density of CCA Replaced Specimens at Different Curing Ages (kg/m^3).

Curing days	**Density at percentage replacement**			
	0% CCA	**5% CCA**	**10% CCA**	**15% CCA**
7 days	2420.88	2461.60	2368.29	2308.70
14 days	2454.00	2499.40	2431.11	2357.00
21 days	2487.77	2530.07	2466.67	2376.29
28 days	2502.00	2564.70	2495.30	2382.80
35 days	2547.50	2587.90	2565.60	2457.77
42 days	2571.25	2572.12	2532.48	2432.29
56 days	2594.07	2606.50	2543.70	2487.11

Table 4
Average compressive strength of concrete mixes.

Mix	**Average compressive strength (MPa) at curing age**						
	7 days	**14 days**	**21 days**	**28 days**	**35 days**	**42 days**	**56 days**
M1	23.40	27.00	30.60	36.00	37.44	38.88	39.96
M2	21.36	24.63	27.23	32.04	33.32	34.60	35.56
M3	19.43	20.12	22.87	26.90	27.97	29.06	29.85
M4	16.43	17.00	19.24	22.64	23.54	24.43	25.13

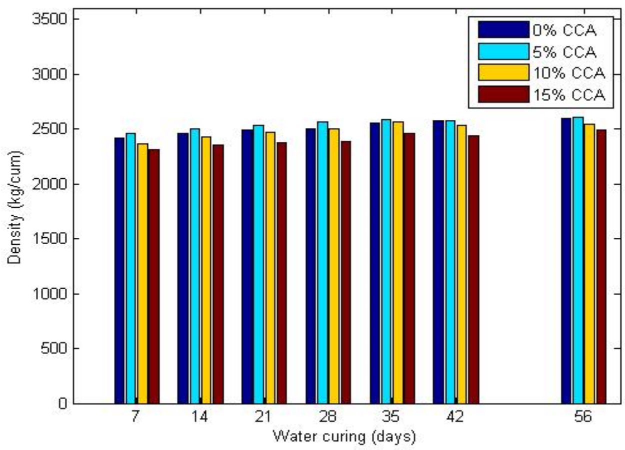

Fig. 2. Density of CCA replaced cubes at different curing ages (kg/m^3).

3.1. Physical/Mechanical Properties:

With the increase in ash content, compared to the cubes without the ash, setting time of concrete was long and water absorption increased. From the obtained results table 2 revealed that the workability decreased as the CCA replacement increased [29]. Table 3 indicated that density increased as curing age increased and it decreased with increasing level of CCA replacement in concrete coupons. Table 4 revealed an increase in the compressive strength of concrete cubes as per increase in curing age and it decreased as per decrease in ash content [5]. Table 4 showed that, in 5% ash replaced concrete cube the compressive strength at 56 day's (35.56 N/mm²) is less than the controlled specimen's compressive strength whose value (39.96 N/mm²) is above the desired compressive strength. However, an important pozzolanic character is the gradual development of characteristic compressive strength [15] which implies that 5% CCA concrete develop the desired strength at 28 days curing age and further increases its strength at later age.

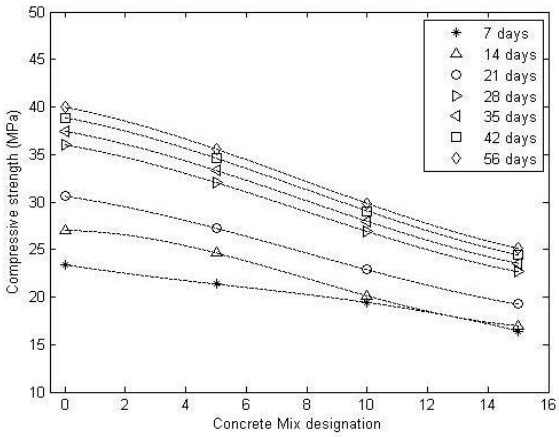

Fig. 3. Compressive strength of CCA embedded Concrete cubes at different ages of curing.

Table 5
Compressive strength of 56 days using Chebyshev's Theorem.

CCA in %	Failure Load in kN	Compressive strength in N/mm²	Mean Compressive strength in N/mm²	Standard deviation of compressive strength	Range of compressive strength in N/mm²
00%	900.000	40.00	39.96	0.4581	37.912 to 42.01
	911.250	40.50			
	886.050	39.38			
05%	795.600	35.36	35.56	0.310	34.17 to 36.95
	810.000	36.00			
	794.700	35.32			
10%	650.250	28.90	29.85	0.820	26.19 to 33.51
	669.375	29.75			
	695.250	30.90			
15%	553.500	24.60	25.13	0.370	23.45 to 26.81
	571.275	25.39			
	571.500	25.40			

In the present study chebyshev's theorem concept and FOSM method were effectively utilized to predict the 95 % probable range of compressive strength of CCA replaced cement concrete of 56 days curing age. Table 5 shows the compressive strength of 56 days concrete cube strength for different CCA replacement ratios. Probable range at standard deviation of 4.47 of compressive strength of concrete cubes of cement replaced with CCA was computed. For control specimen the mean compressive strength was 39.96 N/mm² and its corresponding standard deviation was 0.4581. Here, for k= 4.47 the probable range falls between 37.912

Mpa to 42.01 Mpa respectively. That is, the probable lower compressive strength is 37.912 Mpa and probable higher compressive strength would be 42.01 Mpa.
Table 6 show the FOSM method results which approximately tally with the practically obtained mean compressive strength of concrete cubes by laboratory programme. Here, for controlled specimen (0 % CCA) the mean compressive strength of cubes is 39.96 Mpa and the mean compressive strength obtained from equation 3 is 39.55 Mpa. Hence the laboratory results match well-with the predicted results.

Table 6
compressive strength of 56 days using FOSM method.

% CCA	Failure load kN	Compressive strength (N/mm^2)	Mean Compressive strength (N/mm^2)	Standard deviation of load (σ_p)	Mean Load (μ_P)	μ_R (N/mm^2)	σ_R (N/mm^2)
0%	900.000	40.00	39.960	10.31	899.10	39.55	0.620
	911.250	40.50					
	886.050	39.38					
5%	795.600	35.36	35.560	07.01	800.10	35.195	0.486
	810.000	36.00					
	794.700	35.32					
10%	650.250	28.90	29.850	18.44	671.63	29.544	0.870
	669.375	29.75					
	695.250	30.90					
15%	553.500	24.60	25.130	08.43	565.43	24.872	0.456
	571.275	25.30					
	571.500	25.40					

4. Concluding Remarks:

From the experimental investigations it is concluded that the slump value for 0% CCA was found to be 215.0 mm while for the 15% CCA was obtained as 58.0 mm. This result shows that the workability of the mixes decreases with the increase in CCA replacement. The CCA demands more water. The increased amount of silica in CCA observes high water content. In the present study the CCA replaced concrete mix became less workable when compared with control mix.
It is noted that the average strength reached by concrete specimens at 7 days of curing was 68.56% when compared to 28 days specimens of all 4 mixes. For 28 days the attained compressive strength was 100%. The average compressive strength obtained for 35 days, 42 days and 56 days was found to be 104%, 108% and 108.43% respectively. From the present study it is clear that the hydration process is slow for CCA blended concrete mix compared to controlled mix, however it was observed that the CCA mixes gained more strength at latter age of curing.
The increase in strength for the 15% CCA replaced specimen was found to be 5%, 9.39% and 10% for 35 days, 42 days and 56 days of curing when compared 28 days curing age respectively. Means with increase in CCA content the concrete specimens of latter ages denoted that, it takes more time to form calcium-silicate-hydrate gel. This is due to the reason

that increases in silica content in CCA leads to slower rate of hydration process. Hence with the increase in CCA content the concrete gains strength at the latter age.
In the mix - M4 the compressive strength has reduced to 37.13 % for 56 days of curing. So it is less recommended to replace the cement content beyond 5% of CCA.
With constant value of k=4.472 here the Chebyshev's theorem has utilized in the present study to find out the range of compressive strength for 95% chance of probability. The lower limit and upper limit of the probable range is 37.912Mpa and 42.01Mpa respectively. The present study results helps the new less experienced design engineers to select the suitable amount of CCA content for replacement with cement without any great efforts.
FOSM method had utilized to find out the mean value of compressive strength of concrete cubes and its respective standard deviation. The obtained result matches well with the practical laboratory values hence it is concluded that the theoretical and laboratory results of compressive strength tally well with each other. And wherever there is a difficulty/complexity to setup and perform experimental program, soft-computing techniques like FOSM and Chebyshev's theorem concepts are effectively and successively adopt to predict the stochastic parameter. The only limitation of the present approach is the requirement of large number of testing samples for knowing the standard deviations of variables involved in the stochastic parameter. It demands huge skilled engineers and financial support. Next, researchers can forecast the other hardened properties of CCA embedded concrete by adopting Artificial Neural Network approach.

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