New Seismic Pushover Procedures using either Floor Enforced-Displacements or Inelastic Dynamic Eccentricities on Irregular Single-Storey R/C Buildings

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ABSTRACT

A numerical example of a torsionally-flexible, R/C, asymmetric single-storey building is presented here to clarify in detail the step by step application of two new documented pushover procedures on single-storey R/C buildings. In order to fully consider the coupling between torsional and translational vibrations of the floor-diaphragm under seismic action, the first pushover procedure uses floor enforced-displacements, while the second one uses lateral static floor forces applied with suitable inelastic design eccentricities (inelastic dynamic plus accidental ones) relative to CM. Both pushover procedures referred to the “Capable Near Collapse Principal reference system CRsec(Isec,IIsec,IIIsec)” of the single-storey building. The floor enforced-translations/rotation and the appropriate inelastic dynamic eccentricities used in the two proposed procedures derive from extensive parametric analysis and are given by tables or suitable equations. The evaluation of both procedures relative to the results of non-linear response history analysis shows that both procedures predict with safety the in-plan displacements of the building.


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1. Introduction

The basic tool for the seismic assessment of buildings established by all contemporary seismic codes is the non-linear static (pushover) method of analysis. According to Eurocode EN 1998[1][2], the lateral static force of the pushover method is applied on the Mass Centre (CM) of each floor which has been previously moved from its nominal in-plan location by the accidental eccentricity. However, this often leads to an underestimation of the real dynamic effects in asymmetric buildings, which are caused by the coupled torsional/translational vibrations of the floor-diaphragm due to the developing inertial torsional moment (about vertical axis) of the building floor, both in linear and in non-linear area [3–7]. In every step of the dynamic response, two lateral forces and a torque are acting at each floor level, the composition of which results to an eccentric location of the floor lateral inertial force relative to the Mass Centre, i.e. a dynamic eccentricity appears. In other words, in the framework of the conventional pushover analysis, the (in-plan) displacement demands of the stiff sides or those of the flexible sides of the building are often underestimated. Also, according to EN 1998-1, three patterns of floor lateral static forces can be used in the framework of pushover analysis: an inverted triangular, an (uncoupled) modal and a uniform pattern. However, in multi-storey buildings, this can also lead to an unsafe estimation of the higher mode effects [8]. Moreover, the P-D effects should also be considered in multi-storey buildings. Additionally, EN 1998-1 does not provide detailed information about the building principal axes and refers to the international literature. Therefore, it is unclear which is the appropriate orientation of the floor lateral static forces in the framework of pushover analysis [3,4].

To address the abovementioned problems, various pushover procedures are developed in the last two decades. These procedures can be divided into two main categories: (a) non-adaptive pushovers that use an invariant load pattern and (b) adaptive pushovers that use a variant load pattern. The first category includes pushover procedures that focus on the contribution of higher/torsional modes to take account of the effects of irregularity in elevation or in plan. The lateral load patterns remain constant throughout the analysis. Such procedures are the modal pushovers [9–13] and conventional pushovers combined with some dynamic spectrum analysis [14–19]. Pushover procedures which use dynamic [3–7] or corrective eccentricities [20–22] for the application of the floor lateral static force are also included in the first category. The second category includes pushover procedures that focus on the progressive damage of the building and its impact on the dynamic response characteristics due to stiffness degradation in the non-linear area [23–27]. The lateral load patterns are successively updated at every step or at few steps of analysis.

Despite of the large number of proposed pushover procedures, the scientific community has not yet reached any concrete conclusions. That’s why the various seismic regulations do not directly recommend the use of any specific procedure. Additionally, the implementation of some of the abovementioned pushover procedures is even more difficult than the non-linear response history analysis (N-LRHA), which is the benchmark method for the estimation of seismic demands. Therefore, there is still room for new suggestions on simpler pushover analysis procedures that can safely estimate the seismic demands of irregular in plan/elevation buildings.
This is the objective of this paper which focus on asymmetric single-storey R/C buildings. In this paper, two new pushover procedures are proposed in order to address the in-plan irregularity issues described in detail above. Both procedures aim directly at the Near Collapse (NC) state, providing that the single-storey R/C building under examination shows sufficient ductility and the possibility of floor plastic mechanism formation has been ruled out. The first pushover procedure uses floor enforced-displacements as the action vector (Displacement-Based pushover). Two enforced-translations along two ideal principal axes and one enforced-rotation about vertical axis are applied with appropriate combinations in order to take account of the spatial seismic action. In the second pushover procedure [3,4], which is a Forced-Based one, the abovementioned dynamic eccentricity is treated in a direct way in order to safely estimate the seismic demands of the flexible and stiff sides of the single-storey building. According to the second proposed procedure, the lateral static forces are applied eccentric to CM at two different points of the floor-diaphragm (per loading direction), using suitable inelastic dynamic eccentricities. Considering the two (+) signs of application of the floor lateral static forces, a total of eight separate pushover analyses are performed along the two ideal principal directions. Finally, the spatial seismic action is fully considered from the sixteen SRSS combinations of the effects of the eight separate pushover analyses. Both proposed pushover procedures refer to an ideal 3D "inelastic principal reference system CR<sub>sec</sub>(I<sub>sec</sub>, II<sub>sec</sub>, III<sub>sec</sub>)" at the NC state, which is called as the "Capable Near Collapse Principal system". Its origin coincides with the inelastic centre of stiffness CR<sub>sec</sub> (intersection of the inelastic principal vertical axis III<sub>sec</sub> with the floor-diaphragm) and the two orthogonal horizontal axes coincide with the inelastic principal axes I<sub>sec</sub> and II<sub>sec</sub> of the single-storey building, where all its structural elements have been provided with their secant stiffness E<sub>sec</sub> at yield.

Therefore, in order to perform the two proposed pushover procedures, the following must be specified: (a) the origin for the application of the floor enforced-displacements or for the measurement of the inelastic dynamic eccentricities, (b) the appropriate orientation of the floor enforced-displacements or of the lateral static floor forces, and (c) the magnitude of the enforced-displacements or of the inelastic dynamic eccentricities. In the current work, the step by step application of the two proposed pushover procedures will be clearly outlined through the seismic assessment of a double asymmetric single-storey building, where the theoretical analysis has been given in Bakalis & Makarios [3,4]. Finally, both proposed procedures will be validated relative to the results of N-LRHA.

2. Methodology

In the framework of the second Author’s dissertation, that is in full progress now, and from the extensive parametric investigation using the response history analysis [3,4,8] as well as by the recently international literature review [20–22], the main conclusions and the proposed methodology are summarized below:

1) In the framework of the proposed pushover procedures, which aim directly at the NC state, it is assumed that all the extreme sections of the structural elements have yielded, i.e. all structural elements (columns, beams, walls and cores) have shown plastic hinges at their
critical end-sections (full Near Collapse state). In this ideal “Capable Near Collapse state”,
the secant lateral stiffness $K_{\text{sec}}$ at yield of the single-storey building is calculated considering
that all structural elements of the non-linear building model have been supplied with their
secant stiffness at yield ($EI_{\text{sec}}$), which is strength-dependent [2]. Therefore, this is the most
suitable non-linear model for calculations at the extreme limit (NC) of non-linear area.

2) Both proposed pushover procedures refer to the ideal “Capable Near Collapse Principal
System $CR_{\text{sec}}(I_{\text{sec}}, II_{\text{sec}}, III_{\text{sec}})$” of the single-storey building, resulting from the
abovementioned non-linear model of conclusion (1). Its origin, which is called as the
“Capable Near Collapse Centre of Stiffness $CR_{\text{sec}}$”, is used as a reference point for
measuring the new inelastic dynamic eccentricities of the proposed Forced-Based pushover
procedure and as the application point of the enforced-translations of the proposed
Displacement-Based pushover procedure. The distance between $CR_{\text{sec}}$ and CM is the
inelastic stiffness eccentricity.

3) The two orthogonal horizontal principal axes of the non-linear model of conclusion (1),
which are called as the “Capable Near Collapse Principal Axes $I_{\text{sec}}$ and $II_{\text{sec}}$”, are the axes of
choice for the orientation of the floor lateral static forces of the Forced-based pushover
procedure or of the enforced-translations of the Displacement-based one.

4) The control of the building torsional sensitivity must be performed in the above model of
conclusion (1). The asymmetric single-story buildings are divided into two categories: (a)
buildings with torsional sensitivity when $r_{I_{\text{sec}}}$ or $r_{II_{\text{sec}}}$ ≤ 1.10 $r_m$ applies and (b) buildings
without torsional sensitivity when $r_{I_{\text{sec}}}$ and $r_{II_{\text{sec}}}$ > 1.10 $r_m$ applies, where $r_{I_{\text{sec}}}$ and $r_{II_{\text{sec}}}$ are the “Capable Near Collapse Torsional Radii” in respect to the axes $I_{\text{sec}}$ and $II_{\text{sec}}$
respectively and $r_m$ is the radius of gyration of the floor-diaphragm.

5a) In the framework of the proposed Displacement-Based procedure, the floor enforced-
translations $\psi_{I_{\text{sec}}}$ and $\psi_{II_{\text{sec}}}$, that are applied on $CR_{\text{sec}}$ along the $I_{\text{sec}}$ and $II_{\text{sec}}$ axes, are calculated from the proposed (mean) values of floor inelastic angular deformations $\gamma_{I_{\text{sec}}}$ and
$\gamma_{II_{\text{sec}}}$, at the location of $CR_{\text{sec}}$, shown in Figure 1 (including trendlines) for three categories
of single-storey buildings. The first category consists of pure Frame buildings (without
walls), the second category consists of pure Wall buildings and coupled (via beams) Wall
buildings, while the third category includes Dual buildings (equivalent to frame or wall
buildings). The values of Figure 1 are given separately for the two torsional sensitivity cases
of step 4 and for various values of normalized static eccentricities $e_{R_{I_{\text{sec}}}}/L_{I_{\text{sec}}}$ or
$e_{R_{II_{\text{sec}}}}/L_{II_{\text{sec}}}$, where $e_{R_{I_{\text{sec}}}}$, $e_{R_{II_{\text{sec}}}}$ and $L_{I_{\text{sec}}}$, $L_{II_{\text{sec}}}$ are the inelastic static eccentricities and the
floor-plan lengths along the inelastic principal axis $I_{\text{sec}}$ or $II_{\text{sec}}$, respectively. It is noted
that the accidental eccentricity is ignored for comparison reasons. The floor inelastic angular
deformation, $\gamma_{I_{\text{sec}}}$ or $\gamma_{II_{\text{sec}}}$, is equal to the ratio $\psi_{I_{\text{sec}}}/H$ or $\psi_{II_{\text{sec}}}/H$, where $\psi_{I_{\text{sec}}}$, $\psi_{II_{\text{sec}}}$ is the (enforced)
translational displacement of $CR_{\text{sec}}$ along the axis $I_{\text{sec}}$ or $II_{\text{sec}}$ and $H$ is the
building height.

Additionally, in order to predict the seismic demands of the building flexible sides according
to the proposed Displacement-based pushover procedure, the (mean) floor enforced-rotation
$\psi_R$ about the vertical axis is shown in Table 1 for the three abovementioned categories of
single-storey buildings and for all static eccentricity and torsional sensitivity cases.
Regarding the seismic demand of the building stiff sides, it is considered that their final translational displacement along $l_{secc}$ or $l_{secc}$ is equal with the corresponding displacement of $CR_{sec}$. Therefore, zero floor rotation is considered for the stiff sides ($\psi_R = 0$).

5b) According to the proposed Displacement-Based pushover analysis, in order to take account of the spatial seismic action, we obtain each floor enforced-translation in one principal direction ($\psi_{l_{secc}}$ or $\psi_{l_{secc}}$) and a simultaneous floor enforced-translation equal to 30% of its full value in the other principal direction ($0.3 \cdot \psi_{l_{secc}}$ or $0.3 \cdot \psi_{l_{secc}}$). Moreover, we consider also the total floor enforced-rotation $\psi_R$ about vertical axis. Considering the ($\pm$) signs of action, sixteen (16) possible combinations may be obtained shown in Tables 2 and 3 along each main principal direction, separately. It is noted that $\psi_R$ is equal to zero in those combinations which increase the ductility demand of the stiff sides of the building and it is considered only in the remaining half combinations which affect the building flexible sides only. The envelope of the results of the sixteen (16) separate enforced-displacement pushovers is considered as an estimation of the seismic demand.

![Diagram](image)

**Fig. 1.** Inelastic angular floor deformation $\gamma_{secc}$ and $\gamma_{l_{secc}}$, at the location of $CR_{sec}$, along the $l_{secc}$ or $l_{secc}$ axes used in the calculation of the enforced displacements $\psi_{l_{secc}}$ and $\psi_{l_{secc}}$ of the proposed Displacement-Based pushover.

<table>
<thead>
<tr>
<th>Torsionally stiff single-storey buildings</th>
<th>Floor angular deformation on $CR_{sec}$: $\gamma_{secc} = \psi_{secc}/H$</th>
<th>Torsionally Flexible single-storey building</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRAME</td>
<td>WALL</td>
<td>DUAL</td>
</tr>
<tr>
<td>0.035</td>
<td>0.030</td>
<td>0.035</td>
</tr>
<tr>
<td>0.030</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>0.025</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>0.020</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>0.015</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>0.010</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>0.005</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Table 1.**
Floor enforced-rotation $\psi_R$ (rad) about vertical axis of the proposed Displacement-Based pushover in order to predict the seismic demand of the flexible sides.

<table>
<thead>
<tr>
<th>Frame buildings without walls</th>
<th>Coupled (or pure) Wall buildings</th>
<th>Dual Buildings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0035</td>
<td>0.0050</td>
<td>0.0040</td>
</tr>
</tbody>
</table>

**Table 2.**
Earthquake Spatial Action of simultaneous floor enforced-displacements, where the displacement along axis $l_{secc}$ is maximized (index sec has been omitted).

Eight (8) enforced-displacement combinations of non-linear static analysis.

<table>
<thead>
<tr>
<th>$&quot;+\cdot\psi_I + 0.3 \cdot \psi_{II} + \psi_R$</th>
<th>$&quot;+\cdot\psi_I + 0.3 \cdot \psi_{II} + \psi_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&quot;+\cdot\psi_I + 0.3 \cdot \psi_{II} + \psi_R$</td>
<td>$&quot;+\cdot\psi_I + 0.3 \cdot \psi_{II} + \psi_R$</td>
</tr>
<tr>
<td>$&quot;+\cdot\psi_I + 0.3 \cdot \psi_{II} + \psi_R$</td>
<td>$&quot;+\cdot\psi_I + 0.3 \cdot \psi_{II} + \psi_R$</td>
</tr>
<tr>
<td>$&quot;+\cdot\psi_I + 0.3 \cdot \psi_{II} + \psi_R$</td>
<td>$&quot;+\cdot\psi_I + 0.3 \cdot \psi_{II} + \psi_R$</td>
</tr>
</tbody>
</table>
Table 3.
Earthquake Spatial Action of simultaneous floor enforced-displacements, where the displacement along axis \( I_{\text{sec}} \) is maximized (index \( \text{sec} \) has been omitted).

Eight (8) enforced-displacement combinations of non-linear static analysis.

<table>
<thead>
<tr>
<th>Combination</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>( +0.3 \cdot \psi_I + \psi_{II} + \psi_R )</td>
<td>( +0.3 \cdot \psi_I + \psi_{II} + \psi_R )</td>
</tr>
<tr>
<td>( +0.3 \cdot \psi_I - \psi_{II} + \psi_R )</td>
<td>( +0.3 \cdot \psi_I - \psi_{II} + \psi_R )</td>
</tr>
<tr>
<td>( -0.3 \cdot \psi_I + \psi_{II} + \psi_R )</td>
<td>( -0.3 \cdot \psi_I + \psi_{II} + \psi_R )</td>
</tr>
<tr>
<td>( -0.3 \cdot \psi_I - \psi_{II} + \psi_R )</td>
<td>( -0.3 \cdot \psi_I - \psi_{II} + \psi_R )</td>
</tr>
</tbody>
</table>

6a) In the framework of the proposed Forced-Based pushover procedure, the floor lateral static forces are applied eccentric to CM, using the inelastic dynamic eccentricities \( e_{\text{stiff}} \) and \( e_{\text{flex}} \) with reference to the “Capable Near Collapse Principal System \( CR_{\text{sec}}(I_{\text{sec}}, II_{\text{sec}}, III_{\text{sec}}) \)”. In this way, two in-plan locations of the lateral static forces are specified, along the axis \( I_{\text{sec}} \) or \( II_{\text{sec}} \), the first one towards the building stiff side and the second one towards the building flexible side. The appropriate values of the inelastic dynamic eccentricities \( e_{\text{stiff}} \) and \( e_{\text{flex}} \) have been determined from statistical processing on the results of an extended parametric analysis and are given through Figure 2 and Eqs. 1-4 (prediction lines with a suitable standard deviation).

For torsionally sensitive (flexible) buildings, i.e. when \( r_{I_{\text{sec}}} \) or \( r_{II_{\text{sec}}} \leq 1.10 \cdot r_m \):

\[
e_{\text{stiff},i} = 0.046 \cdot e_{R,i} - 0.11 \cdot r_m
\]
\[
e_{\text{flex},i} = 0.84 \cdot e_{R,i} + 0.12 \cdot r_m
\]

For torsionally insensitive (stiff) buildings, i.e. when \( r_{I_{\text{sec}}} \) and \( r_{II_{\text{sec}}} > 1.10 \cdot r_m \):

\[
e_{\text{stiff},i} = 0.043 \cdot e_{R,i} - 0.05 \cdot r_m
\]
\[
e_{\text{flex},i} = 0.83 \cdot e_{R,i} + 0.17 \cdot r_m
\]
In Eqs. 1-4, \( e_{R,i} \) is the distance between CR\(_{sec}\) and CM in the floor-plan, i.e. the inelastic static (or stiffness) eccentricity along the examined \( i \) direction, which is the horizontal direction \( l_{sec} \) or \( l_{sec}^{II} \).

6b) When the accidental eccentricity is also considered, the proposed Forced-Based pushover procedure is applied using the inelastic design eccentricities. The design eccentricities combine the inelastic dynamic eccentricities (Eqs. 1-4) with the accidental ones, in such a way, that the final location of the floor lateral static forces to be more eccentric relative to the CM in-plan location. The “Capable Near Collapse Centre of Stiffness” CR\(_{sec}\) is again the origin for the measurement of the design eccentricities along the “Capable Near Collapse Principal Axes” \( I_{sec} \) and \( II_{sec} \), with positive direction towards CM (see Figure 9). Therefore, the inelastic design eccentricities \( e_1, e_2 \) (Eqs. 5-6) are used for the application of the lateral loading along axis \( II_{sec} \) while the inelastic design eccentricities \( e_3, e_4 \) (Eqs. 7-8) are used for the application of the loading along axis \( I_{sec} \).

\[
\begin{align*}
   e_1 &= e_{\text{flex},I_{sec}} + e_{a,I_{sec}} \\
   e_2 &= e_{\text{stiff},I_{sec}} - e_{a,I_{sec}} \\
   e_3 &= e_{\text{flex},II_{sec}} + e_{a,II_{sec}} \\
   e_4 &= e_{\text{stiff},II_{sec}} - e_{a,II_{sec}}
\end{align*}
\]

where \( e_{\text{stiff},I_{sec}} \), \( e_{\text{flex},I_{sec}} \) and \( e_{\text{stiff},II_{sec}} \), \( e_{\text{flex},II_{sec}} \) are the inelastic dynamic eccentricities (Eqs. 1-4) along the examined principal directions \( I_{sec} \) and \( II_{sec} \) respectively and \( e_{a,I_{sec}} \), \( e_{a,II_{sec}} \) are the accidental eccentricities. According to EN1998-1, the accidental eccentricities along the principal directions are calculated by the equation \( e_a = \pm (0.05 \sim 0.10) \cdot L \), where \( L \) is the maximum floor-plan dimension normal to the loading direction.

6c) The application of the floor lateral static forces according to (6a) or 6(b), with two signs \( (\pm) \) of action, leads to eight separate pushover analyses to be performed. The target displacement to be reached in each one of the eight separate pushover analysis can be calculated from Annex B of EN 1998-1. The control node coincides with the in-plan location of the applied lateral static forces, i.e. the location determined by the inelastic dynamic or design eccentricities, which is different from the in-plan location of CM. It is noted that the capacity curve(s) of the single-storey building, along the horizontal axis under consideration (\( I_{sec} \) or \( II_{sec} \)), is given by the corresponding base shear and the displacement of the control node, where the lateral static floor force is applied. To take account of the spatial seismic action, the results of the eight separate pushover analyses are combined according to the SRRS rule (sixteen loading combinations), as proposed by Eurocode EN 1998-1 [1]. The envelope of the displacement/deformation results of the previous combinations can be considered as an estimation of the seismic demands.

A flowchart that shows the application steps of the proposed Displacement-Based and Forced-based pushover procedures is presented in Figures 3 and 4, respectively. The abovementioned (2), (3) and (4) are calculated as follows:
a) From a first temporary linear analysis with a static unit floor moment $M_z = 1.00$ kNm about vertical axis, we calculate the lateral displacements $u_{x,CM,M_z}$, $u_{y,CM,M_z}$ of CM along the x and y-axis and the diaphragm rotation $\theta_{z,M_z}$ about z-axis. The coordinates $x_{CR,sec}, y_{CR,sec}$ of the “Capable Near Collapse Centre of Stiffness” $CR_{sec}$, relative to CM, are calculated from Eqs. 9a, b [27,28]:

$$x_{CR,sec} = -u_{y,CM,M_z}/\theta_{z,M_z}, \quad y_{CR,sec} = +u_{x,CM,M_z}/\theta_{z,M_z} \quad (9a,b)$$

b) From a second temporary linear analysis with a lateral static unit force $F_x = 1.00$ kN located on $CR_{sec}$ along x-axis, we calculate the lateral displacement $u_{x,F_x}$ of $CR_{sec}$ along the x-axis. Additionally, from a third temporary linear analysis with a lateral static unit force $F_y = 1.00$ kN located on $CR_{sec}$ along y-axis, we calculate the lateral displacements $u_{y,F_y}$ and $u_{x,F_y}$ of $CR_{sec}$ along the y and x-axis, respectively. The orientation angle $a$ of the horizontal “Capable Near Collapse Principal Axes” $I_{sec}$ and $II_{sec}$, relative to the x , y axes respectively, is calculated from Eq. 10 c[27,28]:

$$\tan 2a = \frac{2u_{x,F_y}}{u_{x,F_x}-u_{y,F_y}} \quad (10)$$

c) From a fourth temporary linear analysis with a lateral static unit force $F_{II} = 1.00$ kN located on $CR_{sec}$ along $II_{sec}$ axis, we calculate the lateral displacement $u_{II,F_{II}}$ of $CR_{sec}$ along $II_{sec}$ axis. Moreover, from a fifth temporary linear analysis, with a lateral static unit force $F_{I} = 1.00$ kN located on $CR_{sec}$ along $I_{sec}$ axis, we calculate the lateral displacements $u_{I,F_{I}}$ of $CR_{sec}$ along $I_{sec}$ axis. The “Capable Near Collapse Torsional Radii” $\eta_{I,sec}$ and $\eta_{II,sec}$ along the $I_{sec}$ and $II_{sec}$ axes respectively are calculated from Eqs. 11a, b [29]:

$$\eta_{I,sec} = \frac{u_{II,F_{II}}}{\theta_{z,M_z}}, \quad \eta_{II,sec} = \frac{u_{I,F_{I}}}{\theta_{z,M_z}} \quad (11a,b)$$

3. Numerical example

An asymmetric single-storey R/C building will be seismically assessed by the proposed pushover analysis procedures for demonstration and validation purposes. In this section, the building characteristics and the non-linear analysis model are described in detail.

3.1 Building characteristics

Figure 5 shows a double-asymmetric, reinforced concrete (R/C), single-storey building, constructed with material grades C25/30 and B500c for the concrete and the steel reinforcement respectively, of average strengths $f_{cm} = 33$ MPa and $f_{ym} = 550$ MPa. The Mass Centre CM lies in the same location with the geometric center of the floor, which is a rigid diaphragm 0.17m thick. The outer perimeter of the floor-diaphragm is formed by cantilevers extending outside of the building layout. The latter is oriented along three different directions. The central part is parallel to the global coordinate system OXY while the left and right parts are inclined by 30°. The structural system of the building is composed by frames and coupled walls. The columns
have a square shape of dimensions 0.45/0.45 m, the rectangular beams are modeled as a Tee section of dimensions 0.30/0.60 m for the beam and 1.60/0.17 m for the flange and the walls have an orthogonal section of dimensions 0.30/1.50 m while the perimeter ones have also one boundary or middle barbell of dimensions 0.45/0.45 m to satisfy the design anchorage length for beam steel bars. The building height is 4 m. The elastic and inertial properties of the non-linear model of the building are presented in Table 4, which also shows the torsional sensitivity check according to step 4 of methodology. The building is characterized as torsionally flexible since both the ratios $r_{l,sec}/r_m$ and $r_{l,lsec}/r_m$ are less than 1.10.

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Fig. 3. Flowchart of the application steps of the proposed Displacement-Based pushover procedure.
Fig. 4. Flowchart of the application steps of the proposed Force-Based pushover procedure.
Table 4.
Elastic, inertial characteristics and torsional sensitivity control of the non-linear model of the building, in which all structural elements have been provided with their secant stiffness $E_{\text{sec}}$ at yield.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static eccentricity $e_{RX}$, $e_{RY}$ (m)</td>
<td>5.28, 1.39</td>
</tr>
<tr>
<td>Static eccentricity $e_{RI\text{sec}}$, $e_{RII\text{sec}}$ (m)</td>
<td>3.01, 4.55</td>
</tr>
<tr>
<td>Norm. static eccentricity $e_{RI\text{sec}}/L_{I\text{sec}}$, $e_{RII\text{sec}}/L_{II\text{sec}}$</td>
<td>0.121, 0.189</td>
</tr>
<tr>
<td>Orientation of $I_{\text{sec}}$ and $II_{\text{sec}}$ axes, relative to x, y axes</td>
<td>-41.85°</td>
</tr>
<tr>
<td>Mass $m$ (tn)</td>
<td>450</td>
</tr>
<tr>
<td>Mass moment of inertia $I_{m}$ (tn∙m²)</td>
<td>45579</td>
</tr>
<tr>
<td>Radius of gyration $r_m$ (m)</td>
<td>10.06</td>
</tr>
<tr>
<td>Torsional radius $r_{I\text{sec}}$ &amp; $r_{II\text{sec}}$ (m)</td>
<td>9.45 &amp; 10.38</td>
</tr>
<tr>
<td>Ratio $r_{I\text{sec}}/r_m$ &amp; $r_{II\text{sec}}/r_m$ (torsional sensitivity)</td>
<td>0.94 &amp; 1.03</td>
</tr>
</tbody>
</table>

Fig. 5. Up: floor plan of single-storey building, down: structural elements sections and reinforcement details.

3.2. Building design

The single-storey building is designed according to the provisions of Eurocodes EN1992-1 and EN1998-1. It is an ordinary building of importance class II ($\gamma_1=1$) and is designed for ductility class high (DCH) with effective peak ground acceleration $a_g = 0.36g$, soil category $D$ and total behavior factor $q=3$. The building is classified into the structural type of dual buildings according to EN1998-1. Specifically, the building is characterized as wall-equivalent dual along the X-
direction and as frame-equivalent dual along the Y-direction. In the design process, all the structural elements of the elastic model of the building have been provided with their effective flexural and shear stiffness that is equal to one-half of their corresponding uncracked (geometric) stiffness. The linear model of the building is also characterized as torsional sensitive ($\eta_{des}/r_m = 0.90$) and the translational uncoupled periods are about 0.21 sec along both the X & Y-axes. Schematic details of the longitudinal and confinement steel reinforcements are presented in Figure 5. Additionally, Tables 5 and 6 show the quantities of steel reinforcement in the structural elements resulted from the design process.

Table 5.
Longitudinal and confinement reinforcement of the vertical resisting elements.

<table>
<thead>
<tr>
<th>Columns &amp; Walls</th>
<th>L. Reinforcement</th>
<th>Conf. Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1, C7</td>
<td>4Ø20+8Ø18</td>
<td>h, b: 4-Ø8/90</td>
</tr>
<tr>
<td>C2, C3, C4, C5, C6, C8, C9, C11, C12, C15, C17, C18</td>
<td>4Ø20+8Ø14</td>
<td>h, b: 4-Ø8/90</td>
</tr>
<tr>
<td>W10</td>
<td>4Ø20+16Ø14+10Ø10</td>
<td>h, b: 4-Ø8/90</td>
</tr>
<tr>
<td>W13, W14</td>
<td>20Ø20+8Ø10</td>
<td>h, b: 4-Ø8/90</td>
</tr>
<tr>
<td>W16</td>
<td>16Ø20+16Ø14</td>
<td>h, b: 4-Ø8/90</td>
</tr>
<tr>
<td>W19</td>
<td>15Ø20+7Ø14+8Ø10</td>
<td>h, b: 4-Ø8/90</td>
</tr>
</tbody>
</table>

* Ø20 means a steel bar of $d=20$ mm and 4-Ø8/90 means four hoops (legs) of $d=8$ mm placed every 90 mm

Table 6.
Longitudinal and confinement reinforcement of beam end-sections (s: start, e: end).

<table>
<thead>
<tr>
<th>Beam section</th>
<th>L. Reinforcement Up</th>
<th>L. Reinforcement Down</th>
<th>Conf. Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>B29s, B38e, B39s, B42e, B43s</td>
<td>5Ø16</td>
<td>5Ø16</td>
<td>h, b: 2-Ø8/90</td>
</tr>
<tr>
<td>B30e, B31s, B33e, B34s, B39e</td>
<td>4Ø16</td>
<td>4Ø16</td>
<td>h, b: 2-Ø8/90</td>
</tr>
<tr>
<td>All other beams start-end sections</td>
<td>4Ø14</td>
<td>4Ø14</td>
<td>h, b: 2-Ø8/90</td>
</tr>
</tbody>
</table>

3.3. Non-linear model

All the structural elements of the non-linear model are supplied with their secant moments of inertia $I_{sec}$ (at their yield). According to EN 1998-3 [2], the secant stiffness $EI_{sec}$ at yield is taken as a constant value over the entire length of each structural element and is equal to the arithmetic average of the $EI_{sec}$ values of its two end cross-sections for positive and negative bending. In the informative Annex A of EN 1998-3, the secant stiffness at yield is given by the equation:

$$EI_{sec} = \frac{M_y}{\theta_y} \cdot \frac{L_v}{3} \quad (12)$$

The chord rotation at yield $\theta_y$ is also calculated by the equations (A.10b) and (A.11b) of EN1998-3 for columns-beams and walls respectively:
\[
\theta_y = \varphi_y \frac{L_v + a_v z}{3} + 0.0013 \left(1 + \frac{15}{L_v} \right) + 0.13 \cdot \varphi_y \frac{d_h f_m}{\sqrt{f_{cm}^3}} \\
\theta_y = \varphi_y \frac{L_v + a_v z}{3} + 0.002 \left(1 - \frac{0.125 L_v}{h} \right) + 0.13 \cdot \varphi_y \frac{d_h f_m}{\sqrt{f_{cm}^3}}
\]

In Eqs. 12-14, \( \varphi_y \) is the curvature at yield, \( M_y \) is the yield moment, \( L_v \) is the shear span, \( z \) is the length of the internal lever arm, \( a_v \) is equal to 1 if shear cracking is expected to precede flexural yielding (otherwise is equal to 0), \( h \) is the depth of cross-section normal to the yield Moment vector, \( d_h \) is the mean diameter of the tension reinforcement, \( f_m \) is the mean yield stress of steel reinforcement and \( f_{cm} \) is the mean compressive strength of concrete. The elastoplastic Moment-Curvature (M-\( \varphi \)) capacity diagrams of all element end-sections are determined by performing section analysis with the module Section Designer of the analysis program SAP2000 [30]. In this process, the axial force of the vertical resisting elements derives from the vertical loads \( G + 0.3Q \) of the seismic combination, where \( G \) is the permanent and \( Q \) is the live vertical load. The shear span \( L_v \) of the structural elements was assumed equal to their half clear length \( L_{cl} \), except the strong direction of walls, the weak direction of internal wall and the direction of columns with cantilever bending, where it was considered equal to \( L_{cl} \). The unconfined and confined model for the concrete follows the constitutive relationship of the uniaxial model proposed by Mander et al.[31]. The steel reinforcement material is represented by the simple model of SAP2000 (R. Park) which is parabolic at the strain-hardening region. Finally, the secant stiffness \( EI_{sec} \) at yield of each structural element is calculated by Eq. 12, along each bending plane, as the arithmetic average of the \( EI_{sec} \) values of element end cross-sections for positive and negative bending. Table 7 shows these \( EI_{sec} \) values as a percentage of the geometric (uncracked) stiffness \( EI_g \), where the average modulus \( E_{cm} \) of concrete C25/30 was considered equal to 31 GPa. In the nonlinear analysis model, interacting \( P-M_2-M_3 \) point plastic hinges are inserted at the end-sections of vertical elements while \( M_3 \) hinges are inserted at the end-sections of beams. The plastic capacity \( \theta_{pl} \) of each end-section, in terms of chord rotations, is determined by the relation \( \theta_{pl} = (\varphi_u - \varphi_y) \cdot L_{pl} \), where the plastic hinge length \( L_{pl} \) is calculated by eq. (A.9) of EN 1998-3.

In Figure 5, we can see the in-plan location of \( CR_{sec} \) and the orientation of horizontal axes \( I_{sec} \) and \( II_{sec} \) calculated according to steps (a) and (b) of methodology. The horizontal axes \( I_{sec} \) and \( II_{sec} \) are rotated by \(-41.85^\circ \) with respect to the Cartesian X, Y axes [27,28]. The inelastic static eccentricities \( e_{R_{I_{sec}}} \) and \( e_{R_{II_{sec}}} \) along the horizontal axes \( I_{sec} \) and \( II_{sec} \) are equal to 3.01 m and 4.55 m, respectively. The building is characterized as torsional sensitive since \( r_{I_{sec}}/r_m = 0.94 < 1.10 \) applies, where the torsional radius \( r_{I_{sec}} \) [29] refers to \( CR_{sec} \) and \( r_m \) is the radius of gyration of the floor-diaphragm (Table 4). The periods of the three coupled modes are \( T_1=0.508 \) sec, \( T_2=0.405 \) sec and \( T_3=0.310 \) sec.

The accidental eccentricity along \( I_{sec} \) and \( II_{sec} \) axes is considered equal to 5% of the maximum plan dimension normal to the loading direction (Figure 10):

\[
e_{a,I_{sec}} = \pm 0.05 \cdot L_{I_{sec}} = \pm 0.05 \cdot 24.92 = \pm 1.25 \text{ m}
\]

\[
e_{a,II_{sec}} = \pm 0.05 \cdot L_{II_{sec}} = \pm 0.05 \cdot 24.15 = \pm 1.21 \text{ m}
\]
where \( L_{\text{sec}} = 24.92 \, \text{m} \) and \( L_{\text{llsec}} = 24.15 \, \text{m} \) are the maximum plan dimensions along \( I_{\text{sec}} \) and \( II_{\text{sec}} \) axes.

### Table 7

<table>
<thead>
<tr>
<th>Columns</th>
<th>( EI_{\text{sec}}/EI_{g} )</th>
<th>( EI_{\text{llsec}}/EI_{g} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1, C3, C4, C5</td>
<td>0.144</td>
<td>0.144</td>
</tr>
<tr>
<td>C2</td>
<td>0.137</td>
<td>0.202</td>
</tr>
<tr>
<td>C6</td>
<td>0.132</td>
<td>0.132</td>
</tr>
<tr>
<td>C7</td>
<td>0.163</td>
<td>0.163</td>
</tr>
<tr>
<td>C8, C12</td>
<td>0.132</td>
<td>0.194</td>
</tr>
<tr>
<td>C9, C11</td>
<td>0.139</td>
<td>0.139</td>
</tr>
<tr>
<td>C15, C17</td>
<td>0.139</td>
<td>0.139</td>
</tr>
<tr>
<td>C18</td>
<td>0.135</td>
<td>0.199</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Walls</th>
<th>( EI_{\text{sec}}/EI_{g} )</th>
<th>( EI_{\text{llsec}}/EI_{g} )</th>
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</thead>
<tbody>
<tr>
<td>W10</td>
<td>0.119</td>
<td>0.168</td>
</tr>
<tr>
<td>W13</td>
<td>0.175</td>
<td>0.322</td>
</tr>
<tr>
<td>W14</td>
<td>0.175</td>
<td>0.322</td>
</tr>
<tr>
<td>W16</td>
<td>0.142</td>
<td>0.303</td>
</tr>
<tr>
<td>W19</td>
<td>0.157</td>
<td>0.323</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Beams</th>
<th>( EI_{\text{sec}}/EI_{g} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>All beams</td>
<td>average value 0.08</td>
</tr>
</tbody>
</table>

\*local axis 3 and 2 are equivalent with the strong and weak structural elements direction.

### 4. Calculation of the enforced-displacements and inelastic dynamic eccentricities

The floor enforced-displacements of the proposed Displacement-Based pushover procedure are determined as follows:

From Figure 1, we take the proposed value of the floor angular deformations \( \gamma_{\text{sec}} \) and \( \gamma_{\text{llsec}} \), at the in-plan location of \( CR_{\text{sec}} \), corresponding to dual torsionally flexible buildings for normalized static eccentricities \( e_{\text{R,sec}}/L_{\text{sec}} = 0.189 \) and \( e_{\text{R,sec}}/L_{\text{sec}} = 0.121 \) and we calculate the enforced-translations \( \psi_{\text{sec}} \) and \( \psi_{\text{llsec}} \) along the axes \( I_{\text{sec}} \) and \( II_{\text{sec}} \), respectively:

\[
\psi_{\text{sec}} = \gamma_{\text{sec}} \cdot h = 0.024 \cdot 4 = 0.096 \, \text{m}
\]

\[
\psi_{\text{llsec}} = \gamma_{\text{llsec}} \cdot h = 0.026 \cdot 4 = 0.104 \, \text{m}
\]

where \( \gamma_{\text{sec}} = 0.024 \), \( \gamma_{\text{llsec}} = 0.026 \) are the proposed floor angular deformations on \( CR_{\text{sec}} \) along the axis \( I_{\text{sec}} \) and \( II_{\text{sec}} \) respectively and \( h = 4 \, \text{m} \) is the height of the single-storey building.

Also, from Table 1 we take the proposed value of floor enforced-rotation \( \psi_{\text{R}} \) about vertical axis for dual buildings, which is used in the half loading combinations of Tables 2 and 3 corresponding to the ductility demand of the flexibles sides, only:

\[
\psi_{\text{R}} = 0.0040 \, \text{rad}
\]

With respect to the proposed Force-Based procedure, the calculation of the inelastic dynamic eccentricities \( e_{\text{stif}} \) and \( e_{\text{flex}} \) (Eqs. 1-2) along each horizontal axis \( I_{\text{sec}} \) or \( II_{\text{sec}} \), as well as of the inelastic design eccentricities \( e_1, e_2 \) (Eqs. 5-6) along the axis \( I_{\text{sec}} \) and \( e_3, e_4 \) (Eqs. 7-8) along
the axis $l_{I\text{sec}}$, which are used for the eccentric application of the floor lateral static forces relative to CM (Figure 9), is performed step by step as follows:

- Stiffness eccentricity ($CR_{sec}$): $e_{R,l_{sec}} = 3.01\, m$, $e_{R,l_{IIsec}} = 4.55\, m$
- Storey Mass: $m = 450\, tn$
- Mass moment of inertia: $J_m = 45579\, \text{tn} \cdot \text{m}^2$
- Radius of gyration: $r_m = \sqrt{J_m/m} = \sqrt{45579/450} = 10.06\, m$
- Min torsional radius: $r_{l_{sec}} = 9.45\, m$
- Torsional Sensitivity: $r_{l_{sec}}/r_m = 0.94 < 1.10 \rightarrow$ Torsional sensitive
- Accidental Eccentricity (Eqs. 15-16) $e_{a,l_{sec}} = \pm 1.25\, m$ and $e_{a,l_{IIsec}} = \pm 1.21\, m$
- Inelastic Dynamic Eccentricities (Eqs. 1-2):
  \[
e_{stiff,l_{sec}} = 0.046 \cdot e_{R,l_{sec}} - 0.11 \cdot r_m = 0.046 \cdot 3.01 - 0.11 \cdot 10.06 = -0.97\, m
  \]
  \[
e_{stiff,l_{IIsec}} = 0.046 \cdot e_{R,l_{IIsec}} - 0.11 \cdot r_m = 0.046 \cdot 4.55 - 0.11 \cdot 10.06 = -0.90\, m
  \]
  \[
e_{flex,l_{sec}} = 0.84 \cdot e_{R,l_{sec}} + 0.12 \cdot r_m = 0.84 \cdot 3.01 + 0.12 \cdot 10.06 = 3.74\, m
  \]
  \[
e_{flex,l_{IIsec}} = 0.84 \cdot e_{R,l_{IIsec}} + 0.12 \cdot r_m = 0.84 \cdot 4.55 + 0.12 \cdot 10.06 = 5.03\, m
  \]
- Inelastic Design Eccentricities (Eqs. 5-8):
  \[
e_1 = e_{flex,l_{sec}} + e_{a,l_{sec}} = 3.74 + 1.25 = 4.99\, m, \text{from CR}_{sec} \text{ to the flexible side of plan along } l_{sec}
  \]
  \[
e_2 = e_{stiff,l_{sec}} - e_{a,l_{sec}} = -0.97 - 1.25 = -2.22\, m, \text{from CR}_{sec} \text{ to the stiff side of plan along } l_{sec}
  \]
  \[
e_3 = e_{flex,l_{IIsec}} + e_{a,l_{IIsec}} = 5.03 + 1.21 = 6.24\, m, \text{from CR}_{sec} \text{ to the flexible side of plan along } l_{IIsec}
  \]
  \[
e_4 = e_{stiff,l_{IIsec}} - e_{a,l_{IIsec}} = -0.90 - 1.21 = -2.11\, m, \text{from CR}_{sec} \text{ to the stiff side of plan along } l_{IIsec}
  \]

5. Seismic assessment

5.1. Seismic DEMAND

In this work, the seismic demand is computed by nonlinear response history analysis (N-LRHA). According to EN 1998-1[1], the mass centre CM is shifted from its nominal in-plan location by combining both accidental eccentricities (Eqs. 15 and 16) along the horizontal axes $l_{sec}$ and $l_{IIsec}$. Considering the four sign combinations (±) of the two accidental eccentricities $e_{a,l_{sec}}$ and $e_{a,l_{IIsec}}$, four shifted in-plan locations of the CM are defined, i.e. four different non-linear models. N-LRHA is performed using three pairs of horizontal accelerograms consisting of five artificial accelerograms created by Seismoartif [32]. The artificial accelerograms (Figure 6) are practically uncorrelated and have similar characteristics with the Hellenic tectonic faults as well as the main specifications of earthquakes recorded in Greece [33]. Each artificial accelerogram has an elastic acceleration response spectrum practically equal to the acceleration design spectrum of EN 1998-1 for soil of Class D (Figure 7). In order to find the most unfavorable loading state, each pair is rotated about the vertical axis successively per 22.5° [34]. Both the horizontal accelerograms of each pair are scaled to a PGA value equal to 0.7g, capable of causing the Near Collapse state of the building. A total of 192 N-LRHA are finally performed and the envelope of the displacements demands along the axes $l_{sec}$ and $l_{IIsec}$ is obtained throughout the floor-plan. This envelope is considered as the "seismic target-displacement" for each control point in the floor-plan.
5.2. Proposed method of pushover analysis

According to the proposed Displacement-Based pushover analysis, the procedure to be performed is illustrated in Figure 8 by applying the sixteen (16) combinations of the floor enforced-displacements of Tables 3 and 4 and finally take the envelope of the results. It is noted that the floor enforced-rotation is used only in those loading combinations (half of them) which affect the building flexible sides.

Also, Figure 9 (left) illustrates the procedure to be performed according to the proposed Forced-Based pushover analysis, which is described below:

1) The floor lateral static forces are applied eccentric to CM, using the inelastic design eccentricities \( e_1 \), \( e_2 \) (Eqs. 5-6) for loading along \( I_{sec} \) axis and \( e_3 \), \( e_4 \) (Eqs. 7-8) for loading along \( I_{sec} \) axis. The origin point for the measurement of design eccentricities is the “Capable Near Collapse Centre of Stiffness \( CR_{sec} \)”. The inelastic design eccentricities were calculated in detail in section 4, as a combination of the appropriate inelastic dynamic eccentricities.
along each horizontal “Capable Near Collapse Principal Axis” $I_{\text{sec}}$ or $II_{\text{sec}}$ (Eqs. 1-2) with the corresponding accidental eccentricities.

Fig. 8. Proposed Displacement-Based pushover procedure with floor enforced-displacements. (left) 8 combinations of enforced-displacements considering axis $I_{\text{sec}}$ as the main principal direction, (right) 8 combinations of enforced-displacements considering axis $II_{\text{sec}}$ as the main principal direction.

Fig. 9. Left: Proposed Force-Based pushover procedure using the inelastic design eccentricities, Right: Capacity Curves according to (up): the proposed Forced-Based pushover analysis, (down) EN 1998-1 pushover analysis.

2) In total, eight (8) pushover analyses are performed considering the two signs ($\pm$) of application of the lateral static floor loads along the horizontal axes $I_{\text{sec}}$ and $II_{\text{sec}}$.

3) The floor-plan displacements resulted from the eight separate pushover analyses, along the $I_{\text{sec}}$ and $II_{\text{sec}}$ axis, are combined with the SRSS rule in order to consider the spatial seismic action. These sixteen (16) combinations are performed at that step of the separate pushover analyses where the seismic target-displacement (N-LRHA) is reached at the monitoring point, which is the application point of the lateral load. An estimate of the seismic demands can be
obtained from the envelope of the (16) SRSS combination results.

For comparison purposes, Figure 10 (left) shows the in-plan location of the lateral static forces according to the EN 1998-1 pushover analysis procedure (N2), i.e. by applying the floor lateral static force on the shifted location of CM by the floor accidental eccentricity. It is worthy note that the in-plan locations of the lateral static forces according to the proposed pushover procedure are in fully disagreement with Eurocode EN 1998-1. In Figure 9 (right), the capacity curves both from the proposed and the EN 1998-1 pushover procedures are also shown.

The sixteen SRSS combinations of the eight separate pushover analyses are as follows: (1) $\oplus$ (5), (1) $\oplus$ (6), (1) $\oplus$ (7), (1) $\oplus$ (8) and (2) $\oplus$ (5), (2) $\oplus$ (6), (2) $\oplus$ (7), (2) $\oplus$ (8) and (3) $\oplus$ (5), (3) $\oplus$ (6), (3) $\oplus$ (7), (3) $\oplus$ (8) and (4) $\oplus$ (5), (4) $\oplus$ (6), (4) $\oplus$ (7), (4) $\oplus$ (8). The numbering of the loading cases is shown in Figures 9 and 10 (1 to 8 in cycle). An estimation of the seismic displacement/deformation demands is calculated from the envelope of these combinations.

6. Results of non-linear methods

The two proposed pushover analysis procedures are validated by comparing the inelastic displacements of the building along the “Capable Near Collapse Principal Axes” $I_{sec}$ and $II_{sec}$
with the seismic demand computed by N-LRHA, which is the benchmark method. Additionally, a comparison with the corresponding results from the EN 1998-1 pushover analysis (N2) and the “corrective eccentricity method” of pushover analysis (Bosco et al. 2017), is also presented. The results are illustrated in Figure 11 and Figure 12 in terms of plan inelastic displacement profile.

First, we notice that the proposed Displacement-Based procedure has enough good estimation near to the building flexible sides but is conservative on the building stiff sides. This overestimation of the building stiff side displacements results in higher seismic ductility demand on this side, fact that has not great importance.

Next, we observe that the pushover analysis according to EN 1998-1 provides unsafe estimates by 18% and 5% for the displacements $u_{I,sec}$ and $u_{II,sec}$ of the stiff sides, along the horizontal axes $I_{sec}$ and $II_{sec}$, respectively. Similarly, the displacements $u_{I,sec}$ and $u_{II,sec}$ of the flexible sides are underestimated by 6% and 4%, respectively.

We also notice that the proposed Force-Based procedure provides safe estimates by 16% and 14% for the displacements $u_{I,sec}$ and $u_{II,sec}$ of the stiff sides, along the horizontal axes $I_{sec}$ and $II_{sec}$, respectively. Also, the displacements $u_{I,sec}$ and $u_{II,sec}$ of the flexible sides are on the safe side by 1% and 4%, respectively.

It is worthy noted that, the “corrective eccentricity” method of pushover analysis (Bosco et al. 2017) predicts with safety the displacements $u_{I,sec}$ and $u_{II,sec}$ of the stiff sides along the $I_{sec}$ and $II_{sec}$ axes, by 8%. On the contrary, the displacements $u_{I,sec}$ and $u_{II,sec}$ of the flexible sides along the $I_{sec}$ and $II_{sec}$ axes are underestimated by 3% and 1% respectively, as in EN 1998-1, because zero “corrective eccentricity” is used in order to estimate the flexible side ductility demands.
Fig. 12. Plan inelastic displacement profile along the “Capable Near Collapse Principal Axes” $L_{sec}$ (left) and $L_{sec}$ (right) resulted from the proposed Forced-Based pushover procedure. Comparison with the seismic demand (N-LRHA) and with EN 1998-1 (N2) and the “corrective eccentricity” (Bosco et al. 2017) pushover procedures.

7. Conclusions

In this paper, two new proposed procedures of documented application of pushover analysis on asymmetric single-story buildings, a Displacement-Based one and a Force-Based one, have been presented. To clarify and evaluate the two proposed procedures, a single-storey R/C building has been assessed. The building is double-asymmetric and torsionally sensitive. The non-linear
model of the building has been formed by providing all structural elements with their secant stiffness $EI_{sec}$ at yield. Then, the following have been determined: (a) the in-plan location of the “Capable Near Collapse Centre of Stiffness” $CR_{sec}$, (b) the orientation of the horizontal “Capable Near Collapse Principal Axes” $I_{sec}$ and $II_{sec}$, (c) the “Capable Near Collapse Torsional Radii” $r_{I,sec}$ and $r_{II,sec}$ relative to the horizontal axes $I_{sec}$ and $II_{sec}$ as well as the radius of gyration $r_m$ of the diaphragm, and (d) the torsional sensitivity of the non-linear model according to the relationship $r_{I,sec}$ or $r_{II,sec} \leq 1.10r_m$. Finally, the inelastic design eccentricities of the new Force-Based pushover procedure, have been calculated from Eqs. 5-8. The latter combine suitable dynamic eccentricities (Eqs. 1-4) with the accidental ones. Also, the floor enforced displacements of the Displacement-Based procedure are determined using Figure 1 and Table 1.

The proposed procedures of pushover analysis, using either the enforced displacements or the inelastic design eccentricities, have been illustrated in detail in Figures 8 and 9. For validation purposes, the in-plan inelastic displacement profile along each horizontal axis $I_{sec}$ and $II_{sec}$ resulted from the application of the proposed pushover procedures has been compared with the seismic demand one computed by N-LRHA. Additionally, a comparison has been made with the inelastic displacement results from the EN 1998-1 (N2) and the “corrective eccentricity” (Bosco et al. 2017) pushover procedures.

The key findings of this investigation and the main conclusions can be summarized as follows:

1) The pushover analysis method according to EN 1998-1 (N2) provides unsafe results for the displacements of the building stiff sides along the horizontal axes $I_{sec}$ and $II_{sec}$. Moreover, the displacements of the building flexible sides along both the horizontal axes $I_{sec}$ and $II_{sec}$ are also a little underestimated. This is due to the incorrect in-plan locations of the lateral static forces, especially as regards the stiff side ductility demands of torsionally flexible buildings.

2) The proposed Displacement-Based pushover procedure (on the numerical examples where have been examined) has enough good estimation near to the building flexible sides but is conservative on the building stiff sides. This overestimation of the stiff side displacements leads to the development of higher seismic ductility demand on this side, fact that has not great importance.

3) The proposed Force-Based pushover procedure predicts with safety the inelastic displacements of the building stiff sides as well as the inelastic displacements of the building flexible sides, along both the horizontal axes $I_{sec}$ and $II_{sec}$.

4) The “corrective eccentricity method” by Bosco et al. [20–22], according to which the floor lateral static load is generally applied less eccentric relative to CM than in the proposed forced-based pushover procedure, also provides safe results for the displacements of the building stiff sides. On the contrary, the displacements of the flexible sides along both the horizontal axes $I_{sec}$ and $II_{sec}$ are a little underestimated, as in the EN1998-1 pushover analysis, since “corrective eccentricity” is not used for the estimation of the flexible side ductility demands, except the accidental eccentricity.

In summary, both the Displacement and Forced-Based proposed procedures of pushover analysis on irregular single-storey buildings, using either enforced displacements or dynamic eccentricities, are simple and effective seismic non-linear static procedures for the safe estimation of the seismic demands due to the coupled torsional/translational response, especially
as regards the displacements of the building stiff sides. The Force-Based pushover procedure of the current paper drives in compatible results with the “corrective eccentricity” method and significantly improves the unreliable and unsafe results of the EN 1998-1 pushover procedure.

References


