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# Approaches in the development of motion sensors in the dynamics of granular media

A.V. Eliseev\* , A.S. Mironov, S.V. Eliseev

Irkutsk State Transport University, 15 Chernyshevsky St., Irkutsk, 664074

Corresponding author: eavsh@ya.ru



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#### **ABSTRACT**

For the processes implemented in the technologies of vibrational strengthening of parts, it is necessary to develop options and justify the constructive and technical schemes of measuring devices that react to the existence of special modes of interaction of the granular working medium with a vibrating surface. Development of methodological positions for the development of measuring devices for evaluating the properties of the vibrational fields of technological machines. Three directions of construction of measuring devices are considered. A signal carrying useful information is formed by changing the sound pressure in the capsule with the microphone. The variants of constructive implementation of schemes for measuring the parameters of the vibrational field are considered. The article uses analytical approaches to the theory of vibrational displacements; it proposes and studies modes with a continuous tossing and the time of approach of the medium element, multiple to the period of the oscillation of the support surface. When processing experimental results, statistical analysis methods are used. A technique for processing the sensor signal is developed and proposed with the use of programs of analysis and detection of regularities. It is shown that the processes of interaction with a continuous tossing have properties that can be reflected in the forms of self-organization of the movement of the elements of the measuring device. The article presents the results of experiments.

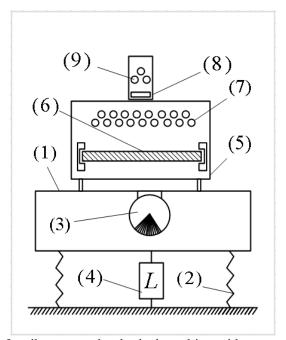
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#### 1. Introduction

The control of technological processes associated with the use of the effects of vibrational interactions is a rather complex problem that is encountered in concrete problems of vibrational transportation, displacement, shock interaction for the purpose of hardening surfaces, etc. [1–3].

Despite a significant number of theoretical and experimental studies [4–8] that are related to the methods of constructing mathematical models with detailed representations about the forms and features of interactions that arise during vibrational contacts, many questions of the study of processes have not yet been properly developed. In particular, this refers to methods and technology for measuring the parameters of continuous technological processes. Some questions of measurement and processing of test results have been reflected in [9–13]. At the same time, controlling the parameters of technological processes in which the physical and mechanical effects of the interaction are reduced to the formation of certain surface properties during continuous collisions, does not always require an accurate knowledge of the quantitative parameters of vibrations, but must provide a number of conditions for certain forms or modes of dynamic interactions of the working medium, the surface of the tool and the workpiece.



**Fig. 1.** Schematic diagram of a vibratory technological machine with a sensor. 1- working body surface, 2-elastic element, 3-source of vibration excitation, 4 - motion transformation device, 5-body, 6-workpiece, 7- "working medium", 8 sensor, 9 "model medium"

To a greater extent, the existence of stable collision processes characterized by signs of dynamic stability during the technological cycle is of interest.

Approaches in the proposed article are developed in relation to the development of sensors of a simplified design, the purpose of which is to visualize the basic characteristics of the vibration process itself, assuming that compliance with such conditions is necessary to ensure the vibration process.

## 2. General provisions. Statement of the research task

The problems of development of measuring means for controlling the parameters of the process of vibration hardening of long products in which the necessary properties of the surface layer are formed in the process of continuous collisions of elements of the granular medium and a vibrating surface are considered.

The model vibration machine (Fig. 1) contains the surface 1 of the working element mounted on the elastic elements 2, the source of vibration excitation 3, the container 5 with the workpiece 6 and the "working medium" layer 7 of the steel balls, the model sensor 9 with the elements of the "model medium "9. Dynamic characteristics of the vibration table are varied by means of the device 4 of the transformation of the movement L.

The object to be treated is rigidly connected to a vibrating surface, and vibrations are created by means of a vibrating stand whose vibration parameters in the course of the technological process must be controlled and adjusted in accordance with the requirements for ensuring the quality of the hardened layer. The loose medium, in this case, is small solid particles (steel balls) forming a layer a few centimetres on the surface of the object being treated.

Comparison of the data obtained on the basis of analytical relationships reflects the circumstances that the change in a small number of factors results in a variety of processes and creates difficulties in obtaining accurate recommendations for the maintenance or adjustment of technological processes. Nevertheless, information on the qualitative diversity of the vibration interaction processes, the forms of trajectories, phase relationships is sufficiently adequate to actual processes and is confirmed by practical experiments. In this regard, a number of issues arise, the solution of which can create certain bases for the development of means for obtaining information on the qualitative characteristics of the process of vibrational interactions in their integration interactions.

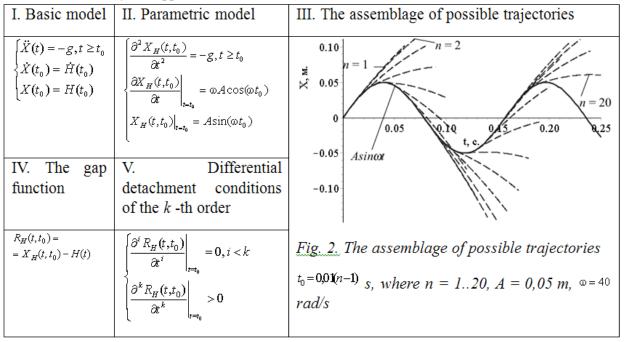
The research task is to develop the principles of construction and methods of designing sensors that fix the effects of vibration processes, reflecting the features of the interaction of the particle layer, the vibrating surface and the workpiece.

#### 3. Mathematical model

The development of the generalized approach is based on the evaluation of the characteristics of the particle trajectories with the formation of the free-flight phase with subsequent interaction with the vibrating surface. The mathematical model of the interaction of a material particle with a horizontal surface oscillating according to a harmonic law  $H(t) = A\sin(\omega t)$  is used as the basis. The characteristics of the interaction of a material particle with a vibrating surface are determined taking into account the adjustment parameters of the basic and extended mathematical models. The basic model is expanded by taking into account additional constant forces, viscous friction, and other factors.

Table 1 presents the main elements of the analytical approach: the basic model, the relationships with the parameters, the assemblage of possible trajectories in the flight phase (Figure 2), the gap function, the detachment conditions.

**Table 1** Elements of the analytical approach.



To comparatively assess the possibilities of influencing the process of parameters, the gap function is used, which makes it possible to conduct a detailed study of the most significant properties of vibration interactions of elements with allowance for non-retentive bonds.

For the variant of the basic model, in which the surface moves according to the harmonic law, the gap function takes the form:

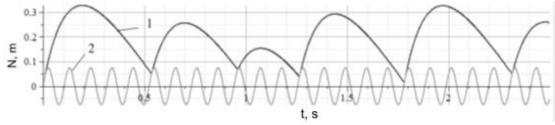
$$R_{H}(t,t_{0}) = A\sin(\omega t_{0}) - A\sin(\omega t) + A\omega(t-t_{0})\cos(\omega t_{0}) - \frac{1}{2}g(t-t_{0})^{2}, t \ge t_{0}.$$
 (1)

A generalized approach based on the use of the gap function involves considering a family of possible trajectories in the free-flight phase. In the general case, the interaction of particles with the surface can occur under the laws of motion of the surface, which have a more complex shape than the harmonic ones.

Within the scope of representations about the gap function, the conditions for the detachment of a material particle with unilateral constraints are determined not only by the position of the particle or by the phase parameters of the harmonic process, but also by certain relationships between displacement, velocity, acceleration, sharpness (which should be understood as the third derivative of displacement, analogous to the concept of smoothness or sharpness movements used in problems of the dynamics of transport devices).

With the help of the gap function, dynamic processes are simulated, steady-state interactions, which are periodic tossing of particles of the working medium; such particles reflect the properties of steel balls a few millimeters in diameter, have a free-flight phase, a partial idling phase, and then resume the cycle. Mathematical models for studying the characteristics of periodic processes of detachment, flight, and bouncing often have a simplified form when the motion of the surface is considered harmonic, and the characteristics of impact interactions reflect the quality of the machined surface of the part.

The most rational from the point of view of the effectiveness of the interaction are the parameters of the processes in which the particles have a minimum time of idling. In such situations, the bouncing of a ball or a material particle proceeds continuously, and the collision processes after the end of the approach phase occur with optimal shock interaction parameters. The features of periodic regimes in which the duration of the free-flight phase is a multiple of the period of the surface oscillations considered in [10]. Fig. 3 shows the characteristic trajectory of a material particle with a shift in several periods of oscillation of the support surface, taking into account the viscous frictional forces between the particle and the medium.



**Fig. 3.** A characteristic trajectory of tossing with flight of a particle through several periods. 1 - trajectory of tossing of the particle. 2 - trajectory of the oscillation surface with amplitude A=0.74 m.,  $^{60}$ =60 rad./s., p=14 kg/s.

The choice of the amplitude and frequency of the oscillation of the reference surface that are compatible with each other in a certain ratio makes it possible to provide a regime of multiple periodic flip-flops (Figure 4).

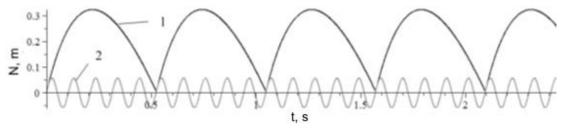


Fig. 4. A particle trajectory at the implementation of the mode with multiple tossing.

Developed mathematical models described in the scientific literature, including in [10], make it possible to evaluate the influence on the parameters of the interaction processes taking into account the action of various factors, among which the resistance of the working medium, the frequency and amplitude of the oscillations, the possibility of interactions that the contacts formed in the structure of the vibrating layer.

At the same time, the mathematical models of the motion of one particle on a vibrating surface are the basis for the development of mathematical models of the dynamics of a granular layer interacting with a vibrating surface.

The development of the theoretical foundations of the dynamics of the granular layer for the development of vibration field control technology [14] assumes the formation of an experimental base of physical effects that can be detected with the help of a number of special sensors developed on unified principles and reflecting the motion of the working mixture of vibrating technological machines on the basis of indirect characteristics.

# 4. Principles of construction and design features of sensors

Existing methods of research and sensors used record the parameters of the vibration process by measuring the displacement, speed, acceleration, sharpness of individual points of the vibrating table or individual points of the workpiece. As for the quality of the workpiece, the idea of this is formed on the basis of experimental data obtained from the comparison of the parameters of vibrational interactions and the results of evaluating the properties of the hardened surface of the workpiece.

The sensor of the boundary parameters. To register critical modes of vibrational interactions with the formation of a discontinuity, an experimental sensor of the boundary parameters has been developed, the principal scheme of which is shown in Fig. 5.: 1-cylindrical body, 2-inertial body, 3-piezosensitive element, 4-fastening element, 5-vibrating base, 6-current leads [15].

The sensor generates a signal giving a representation in the process of oscillation about possible forms of the discontinuity of contacts of elements of the working medium with a vibrating surface based on the movement of a single characteristic "representative" of the working mixture. With the help of such a sensor, single-tossing modes, continuous-tossing modes, multiple-tossing modes, etc. can be defined. The installation of several sensors of the boundary parameters at several control points of the working body of the vibration table makes it possible to evaluate the structure of the vibrational field.

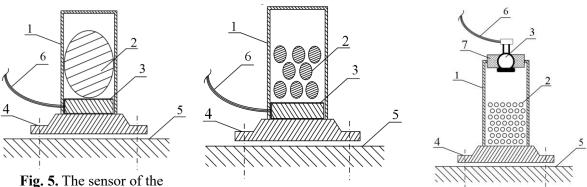
At the same time, the possibilities of the sensor of the boundary parameters are limited by the number of elements of the medium inside the housing. The motion of a single element in the process of interaction with a vibrating surface is fundamentally different from the motion of two, three, etc. elements that, while inside the sensor housing, interact with the oscillation surface and with each other.

The sensor of the dynamic states evaluation. To account for the interaction of several elements, an experimental sensor for evaluating dynamic states is proposed (Fig. 6). An experimental sensor is mounted on the 5 (surface of the vibrating table) and contains 1 (cylindrical housing); 2 (inertial elements of the working medium); 3 (piezoelectric element); 4 (fastening element); 6 (current leads). The body of the dynamic state estimator contains several "representatives" of the working medium of the vibrating technological machine. The sensor device, in turn, assumes a large "resolution" for recording the physical effects occurring within the working layer.

At the same time, "information" on the interaction between the elements of the "model medium" of the sensor is transmitted only as a result of contacting certain elements of the medium with the piezoelectric crystal located at the base of the enclosure. The interaction between the elements is transmitted indirectly through the intermediate elements to the piezoelectric crystal. In turn, the process of interaction between all elements of the layer in integral form is of interest.

The sensor of integral evaluation of the dynamic state of the medium. To record the integral characteristics of the interaction of elements of the working medium on the basis of indirect indicators, a sensor for the integral evaluation of the dynamic states of the medium has been developed.

Fig. 7 shows the scheme of the integral state estimation of the dynamic state, containing 1 (a cylindrical body), 2 (elements of the working medium (metal balls)); 3 (microphone); 4 (fastening of the cylindrical sensor on the bearing surface of the vibration table); 5 (bearing surface of the vibration table); 6 (microphone current leads); 7 (the microphone holder).



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**Fig. 6.** The sensor for assessing the dynamic states of the working medium.

**Fig. 7.** The sensor of integral estimation of dynamic states of the working medium.

The sensor for integral evaluation of dynamic states (Fig. 7) works as follows: with periodic disturbance from the vibrating base 5 of the vibrating technological machine, movement of the inertial elements 2 of the "model medium" occurs, accompanied by shocks, displacements, floppings, which causes oscillations of the air column inside the sensor housing, transferred to the membrane of the microphone 3. The signal from the microphone through the current leads is transferred to the analog-digital converter.

Depending on the shape of the oscillation of the supporting surface of the vibrating technological machine, a signal is generated that determines the mode of movement of the bulk working medium of the vibrating technological machine. The steady-state motion of the elements of the working medium 2 forms a detailed signal at the current leads. The registration of the signal corresponding to the oscillation of the elements of the working medium allows the signs of the current operating mode of the vibrating technological machine to be fixed. The installation of several sensors in the control points of the working body allows you to record the peculiarities of the vibrating field of the technological machine.

The proposed designs of sensors are based on the assumption of the similarity or interrelation between the dynamic characteristics of the "working medium" of the vibrating technological machine and the "model medium" of the sensor placed inside the sensor housing. The inertial elements of the "model medium" of the sensor are identical to the elements of the "working medium" of the vibrating technological machine. At the same time, the question of the identity of the dynamics of the "model medium" and the "working medium" is open. The use of sensors involves two methods. The first way is to establish a relationship between the motions of the "model medium", the "working medium" and the forms of oscillation of the working surface. The establishment of interrelations or similarity between the indicated processes of media motion and the surface allows us to determine unknown factors via known ones. The second method implies that the movement of the "working medium" and the surface is unknown, but can be measured with the motion of the "model medium".

The essence of the method is that a certain mode of motion of the "working medium" is declared a priori as a "reference" mode. During operation of the system in the "reference" mode, a signal is collected from the sensors, which acts as a basis for the future comparison. The efficiency of the sensor is determined by how much it qualitatively reflects the dynamic processes of interaction between the "working medium" and the surface of the working organ.

"Model medium" and the device of the sensor are a factor in the formation of qualitative signs of the signal reflecting the dynamic characteristics of operation processes.

#### 5. Justification of the technical result

To substantiate the possibility of obtaining a technical result, it is proposed to use methods of mathematical statistics that, on the basis of the homogeneity criterion, can verify the statistical significance of the difference in signals from sensors into the structure of which additional elements are introduced. To confirm the possibility of obtaining a technical result, a prototype sensor was developed and a comparative experiment was performed on a laboratory vibration installation with the signal being transmitted to an analog-to-digital converter. Signals I from the prototype sensor and signals II from a specially modified sensor were recorded on a laboratory vibration table, operating at a frequency of 20-30 Hz with an amplitude of no more than 1 mm are recorded. Signals I and signals II have spectral representations in the form of graphs of functions depending on frequency (Fig. 8). The presented coefficients in the Fourier expansion of signals I and signals II indicate the difference of signals, but the statistical significance of their differences requires confirmation.

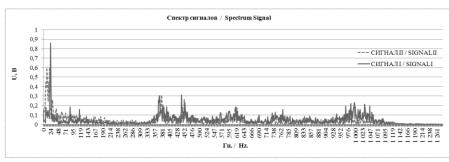
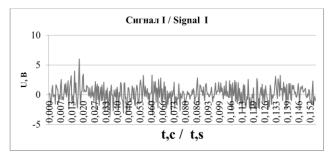


Fig. 8. Spectral representations of signals I and signals II.

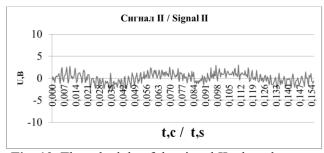
In Fig. 9 is a graph of the signal I recorded by the recording equipment for the case where the metal balls are placed inside the sensor housing of the integral evaluation of dynamic states (Fig. 7). Figure 10 shows a graph of the signal II recorded by the recording equipment for the case where there are no metal balls inside the sensor housing. Using the prototype sensor, an experiment was performed to confirm or deny that the prototype sensor, taking into account the imposition of noise and external noise, registers a useful signal that records the interaction of inertial elements of the "model medium" of the sensor.

The magnitudes of signals I and signals II at discrete instants of time are considered as implementations of random variables with some unknown continuous distribution. Based on the results of the laboratory vibratory installation, samples X and Y of m and n sizes n = 32768 were generated based on signals I and signals II.

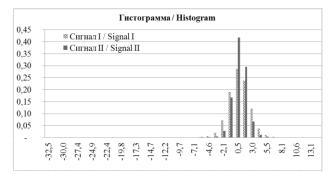
The fact that the useful signal was recorded was confirmed by the statistical significance of the difference between the signal I recorded from the sensor with inertia elements (Fig. 9) and the signal II recorded from the sensor without inertia elements (Fig. 10), based on Smirnov's homogeneity criterion [16] with a level of significance  $\alpha = 0.05$ . Fig. 11 shows the histograms of relative frequencies for samples X and Y.



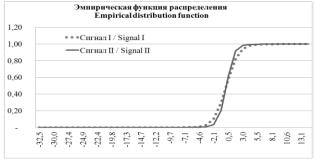
**Fig. 9.** The schedule of the signal I when metal balls are placed in the sensor case.



**Fig. 10.** The schedule of the signal II when there are no metal balls in the sensor case.



**Fig. 11.** The histogram of relative frequencies for a signal I and a signal II.



**Fig. 12.** Empirical functions of distribution for signals I and signals II.

The Smirnov homogeneity criterion (the Kolmogorov-Smirnov test) showed that the  $H_{\theta}$  hypothesis is rejected ( $H_{\theta}$  is the hypothesis of the homogeneity of signals I and signals II, the

corresponding statistics  $S = \sqrt{mn/(m+n)} \cdot D_{m,n} \approx 2.87$  exceeds the critical value  $S_{0.05} = 1.36$  corresponding to the significance level  $\alpha = 0.05$ , m, n are the sample sizes,  $D_{m,n} = \sup \left| F_n^*(y) - G_m^*(y) \right|$  is the characteristic of the difference between the empirical distribution functions  $F_n^*(y)$ ,  $G_m^*(y)$ , constructed by reference to samples X and Y).

The rejection of the hypothesis of homogeneity indicates the statistical significance of the difference between signals I and signals II, which is interpreted as the possibility of obtaining a useful signal of the "model medium" of the sensor characterizing the dynamic qualities of the "working medium" of the vibrating technological machine. The function of the relative frequencies and the corresponding distribution reflects the characteristics of the structure of the vibrational field.

At the same time, situations are possible where sequences of measurements taken from a single process of operation of a vibration machine, but fixed at different time intervals, are different in the sense of statistical significance.

Table 2 presents data reflecting the statistical significance of the differences between the sequences of signals recorded at time intervals from the oscillation processes of sensors with 1,2, ... 7 elements.

Table 2 shows that signal sequences from different time intervals of a single workflow with a fixed number of "model medium" elements can be different in a statistical sense using the Smirnov (Kolmogorov-Smirnov) homogeneity criterion.

**Table 2** Statistics for check of a hypothesis of uniformity of signals.

N quantity of balls in the	S								
balls in the housing of the sensor, pcs.	Hypothesis $H_0$ for signals 1 and 2	Hypothesis $H_0$ for signals 1 and 3	Hypothesis $H_0$ for signals 2 and 3						
N=1	4,458	4,308	0,681						
N=2	3,855	2,195	4,370						
N=3	0,908	6,350	6,722						
N=4	0,958	3,580	3,745						
N=5	2,802	2,802	1,356						
N=6	1,836	2,052	0,805						
N=7	0,973	1,409	1,075						

During the experiment, three intervals were recorded from processes with a fixed number of elements. For each process, the relevant statistics are calculated to check between three pairs of

three signal sequences. The Smirnov homogeneity criterion makes it possible to test the hypothesis of homogeneity. In particular, the calculation of statistics (Table 2) shows that during the operation of a laboratory vibration table, the readings of the sensor with a fixed number of elements of the "model medium" can be statistically different. In this case, the work of the laboratory vibration table with conditionally "identical" parameters generates signals that are non-uniform, in the statistical sense.

At the same time, it should be noted that the statistical distinguishability of signals from sensors with different number of elements of the "model medium" is significantly higher than the statistical "distinguishability" of signals from processes that are generated by sensors with a fixed number of elements of the model medium.

Table 3 shows the values of statistics corresponding to Smirnov's homogeneity criterion. For each  $N=1,2\ldots 7$  signals were recorded and corresponding statistics were calculated for each signal. The hypothesis of homogeneity is rejected for all pairs of signals, since the structure of the vibrational field can be represented or reflected by a spectral characteristic, an empirical distribution function, a polygon of relative frequencies.

When recording the characteristics of the vibrational field, it becomes possible to develop additional methods for varying the vibrational field.

**Table 3**The statistical importance of distinctions of signals from sensors.

N quantity of balls in the housing of the							
sensor, pcs.	N=1	N=2	N=3	N=4	N=5	N=6	N=7
N=1		18,53	26,75	24,15	7,53	13,88	5,85
N=2	18,53		7,88	5,10	17,63	2,02	5,11
N=3	26,75	7,88		4,33	25,51	7,96	12,11
N=4	24,15	5,10	4,33		22,58	3,96	5,98
N=5	7,53	17,63	25,51	22,58		18,61	8,83
N=6	13,88	2,02	7,96	3,96	18,61		4,17
N=7	5,85	5,11	12,11	5,98	8,83	4,17	

# 6. Registration of the forms of motion of elements in model problems depending on the parameters

The signal generated by the sensor based on the motion of the "model medium" depends on the characteristics of the elementary "representatives" of the medium and their number. By selecting the number of elements of the "model medium" and the shape of the sensor housing, it is possible to obtain a signal with different characteristics, which, in particular, can be the mean

value or the standard deviation from the mean. The features of functional dependences between the main characteristics of the signal are of interest.

In Fig. 13 and Fig. 14 there are graphs of typical signals from sensors with 2 and 22 elements of "model media", respectively. The presented variants of the signals represent the process, conditionally called "quasi-beats", characterized by a periodic increase in the oscillation amplitude of the control points of the model vibration table in accordance with Fig. 13 and Fig. 14. The same technological process is reflected by different graphs of signals from sensors of one basic design, but with different numbers of elements of the "model medium".

Fig. 15 and Fig. 16 show the histograms of the functions of mean values and root-mean-square deviations for sensor signals when the number of elements of the model medium changes from 0 to 27 (the data are averaged).

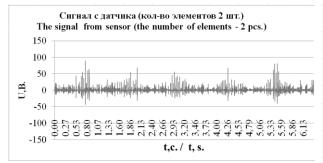


Fig. 13. The signal is written down from the sensor which contains 2 balls in the housing.

**Fig. 14.** The signal is written down from the sensor which contains 22 balls in the housing.

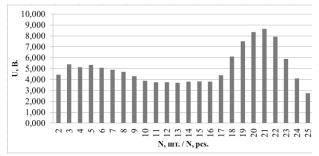
Fig. 15 and Fig. 16 show that for separate certain parameters the graphs of the functions have local maxima and minima. The value of parameters delivering maxima to certain characteristics is of particular interest, since the sensitivity of the signal from the sensors for the dynamic process increases. the local minima as special states are also of interest.

The presented graphs on the basis of the conducted experiments reflect the relationships between physical processes and create prerequisites for determining the analytical patterns of the dynamics of elements of granular media.

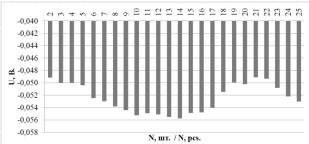
#### 7. Conclusion

On the basis of the conducted studies related to the review and evaluation of measurement methods, it can be asserted that during the creation of measuring equipment for vibrating technological machines the attention is primarily given to recording and monitoring the current parameters of the vibration process: displacement, vibration velocities and vibration acceleration. However, for specific technical applications, such as vibratory hardening, it is more interesting to adhere to the modes of continuous tossing. For all the development of mathematical models, the transitions from the consideration of interactions of one material particle with a vibrating surface to representations of what processes arise when a layer of granular medium interact with a

surface being treated are realized (represented) very difficult. The output can be found by using sets of sensors in which the vibrational interaction process is actually reproduced.



**Fig. 15.** Averages of signals from sensors depending on N - quantity of elements of an inertial body (averaging on the next points).



**Fig. 16.** A mean square deviation of signals from sensors depending on N – quantity of elements of an inertial body (averaging on the next points).

Traffic features recorded by a cluster of sensors can be used to build a system for monitoring the process of surface modification as a result of vibrational interactions. The forms of signals reflecting the dynamics of the motion of granular media are determined by the device of sensors, which to some extent reproduce the global movement of the "working medium" through the motion of the "model medium".

The constructive diversity of sensors generates a variety of signals reflecting the dynamic processes of interaction of the "working medium" with the vibrating surface of the technological machine. The "model medium" and sensor design serve as factors that form the relationships between the dynamic interaction of the "working medium" of the vibrating machine and the "model medium" of the sensor. To determine the significant structural factors of the interrelation between the "model medium" - "working medium" -the vibrating surface, the method of determining the statistical significance is used. The number of elements of the "model medium" is an important factor affecting the quality of recording dynamic effects that are implemented in the "model medium", reflecting the effects of the dynamics of regimes in the process of the vibrational technological machine.

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