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Elastic Buckling of Single-Stepped Columns with End and Intermediate Axial Loads

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ABSTRACT

Buckling study of single-stepped columns subjected to both intermediate and end axial loads are introduced in this paper. The column understudy is considered as two segments where the upper and the lower parts have different cross section moment of inertia or different material and subjected to intermediate load at the location of the cross section change beside the end load. All the classical end conditions of the studied column are considered in this paper as pinned ends, clamped ends, clamped-free ends and clamped-pinned ends. The analysis is developed using finite element method to study the effect of each parameter may be affect in the buckling loads. These parameters are i) ratio of the intermediate axial load to the end axial load, ii) the intermediate load location as a ratio to the column span and, iii) the ratio of flexural rigidity of lower segment to that of upper one. The obtained numerical results are introduced in many interaction curves to obtain the buckling loads for each end conditions considering the other parameters. A comparison between the obtained results and that of the available theoretical studies shows the accuracy and the simplicity of the present work to get the critical load.

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Introduction

The critical buckling load of columns and its behaviour is an important item in the different structures study. Earlier, Leonhard Euler obtained the critical buckling load for pinned ends column with uniform cross section which frequently termed as the Euler load. Columns with uniform cross section are not the most economical form to be stable against buckling. In many applications of the civil engineering, stepped columns may be required from the view of the economical design especially when the column subjected to intermediate and end axial loads. Examples to this loading case are crane columns in industrial buildings and columns supporting intermediate floors. The case of a two segment column with pinned ends compressed by end and intermediate axial forces was studied by Timoshenko and Gere [1]. Exact buckling loads for columns with uniform cross section under the effect of intermediate and end axial loads have been derived by C. M. Wang and I. M. Nazmul [2]. They divide the column to two segments and the differential equations for each segment are investigated and solved together to get the stability criterion. Wilson [3] used a finite difference scheme to represent the fourth order differential equation for the stepped column under end axial load only to get an approximate buckling load. Salama [4] introduced a theoretical analysis of the stability of stepped column under end and intermediate axial loads using the potential energy method considering pinned ends and clamped-free ends only. The buckling problem of two portions stepped column is developed by Pinarbasi and et. [5]. They solved the derived differential equations by using the variational iteration method (VIM). In this paper, stability study of two segment stepped columns under the effect of combined axial loads are developed using finite element method considering different combination of end conditions and the results are compared with other studies.

2. Theoritical analysis

2.1. Assumptions

Consider a stepped column as shown in Figure (1-a) subjected to end axial load P_1 at top end and an intermediate axial load P_2 at a distance $x = \alpha$ L from the bottom with the following assumptions:

a) b)

Fig. 1. a) Stepped column subjected to end and intermediate axial loads, b) Finite element model.

i. The used material is linearly elastic.
ii. No initial imperfection of the column.
iii. No local buckling at any cross-section along the column length is allowed.
iv. The moments of inertia of the upper and lower parts are I_1 and I_2 respectively.

2.2. Method of Analysis

Firstly, the stability of the stepped column under end and intermediate loads depends on the following ratios

i. Ratio of the intermediate axial load to the end axial load $m = (P_1+P_2)/P_1$.
ii. The intermediate load location as a ratio to the total column length α.
iii. Ratio of moment of inertia of lower segment to that of the upper segment $n = I_2/I_1$.

The buckling load can be expressed by the following formula

$$(P_1 + P_2)_{cr} = \mu \frac{\pi^2 \, E \, I_1}{(k \, L)^2} \tag{1}$$

Where

E denotes the modulus of elasticity of column material,

μ denotes the buckling load coefficient that depends on the ratios m, n and α, *and*

k denotes end condition parameter for the uniform column under end axial load

$k = 1.00$ ---------------- for pinned ends column [P-P]

$k = 0.50$ ---------------- for clamped ends column [C-C]

$k = 0.699$ -------------- for clamped-pinned ends column [C-P]

$k = 2.00$ ---------------- for clamped-free ends column [C-F]

Assuming values for the ratios m, n and □□for each particular case, the column under study is modelled as a three dimensions frame element as shown in Figure (1-b) with very large number of elements. The classical end conditions are considered in the models under study. The critical buckling loads for the considered column have been obtained using SAP2000 program based on the finite element method.

Mode shapes for different values of the location of the intermediate load (α) (for C-C column as an example) are shown in Figure (2) that describe the buckling behaviour of the considered column.

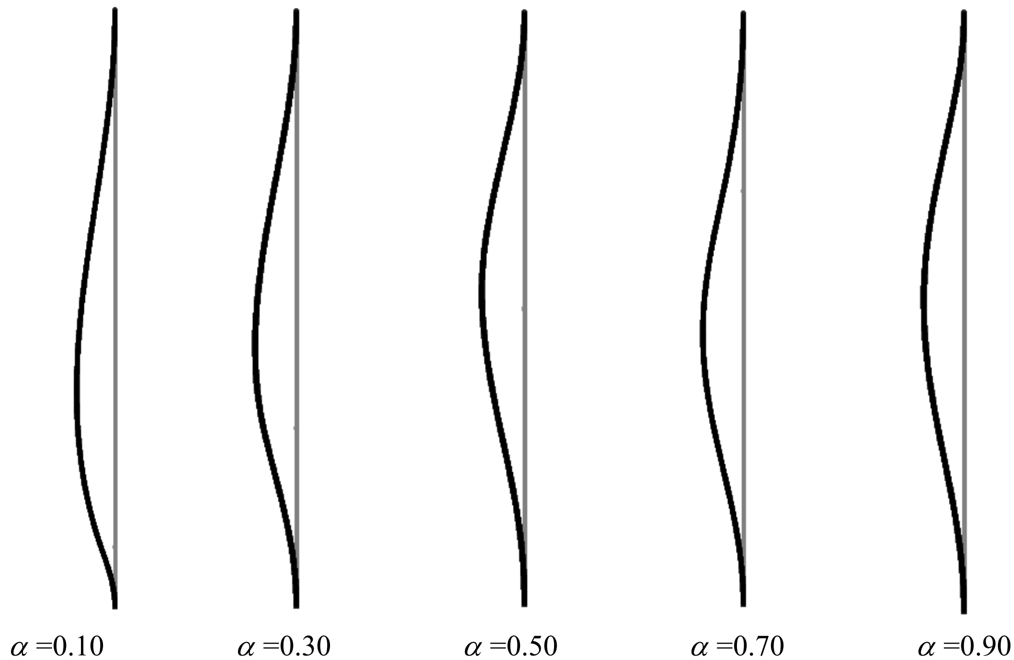


Fig. 2. Effect of the intermediate load location in the buckling shape of the stepped column (C-C column)

To study the interaction relation between the buckling loads P_1 and P_2, equation (1) can be rewritten in the following form

$$\left(P_1\right)_{cr} = A_1 \frac{\pi^2 \, E \, I_1}{(k \, L)^2} \quad , \quad \left(P_2\right)_{cr} = A_2 \frac{\pi^2 \, E \, I_1}{(k \, L)^2} \tag{2}$$

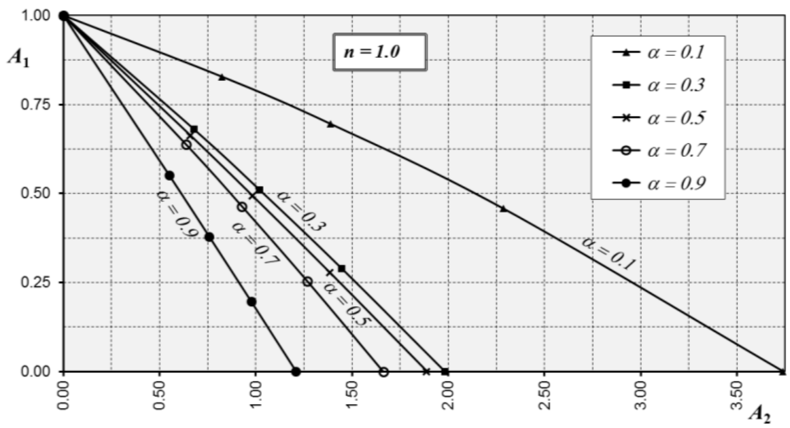
Where, A_1 and A_2 are the buckling factors for the end and intermediate loads that can be expressed as follows

$$\mu = A_1 + A_2 = m \, . \, A_1$$

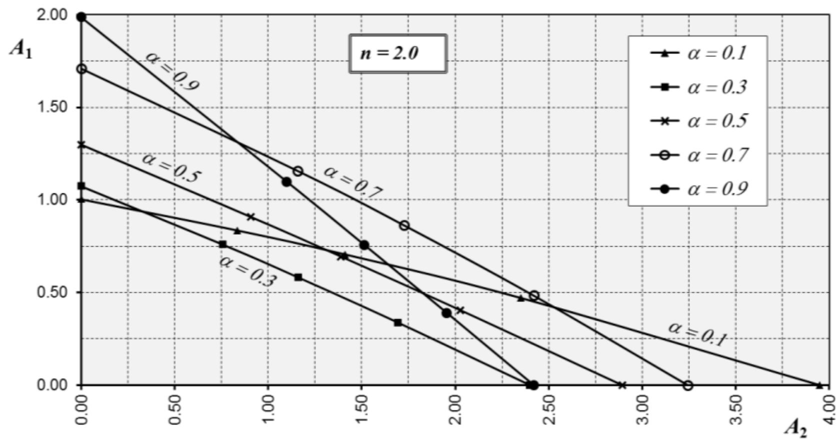
Generally, each one of these factors depends on the other and the increasing of one of them causes the other to decrease. The relation between these factors is obtained by numerical analysis using finite element method for a certain value of the ratios n and α.

3. Results and discussion

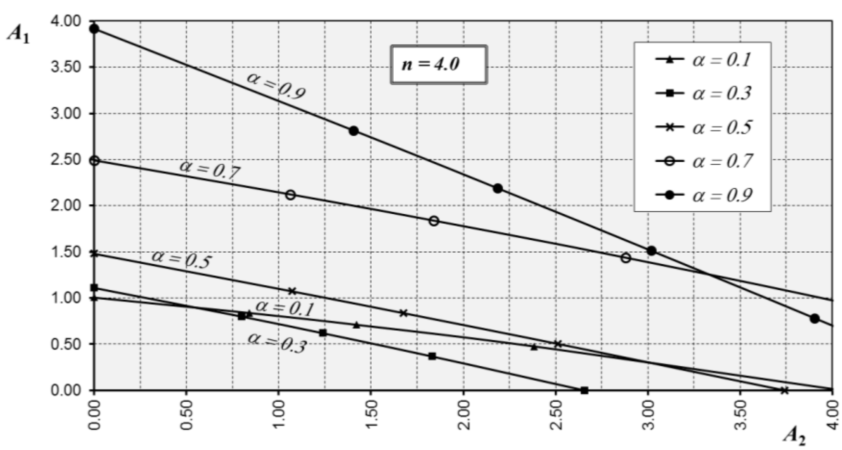
Finite element analysis solution for two-segment stepped columns subjected to both intermediate and end axial loads are presented in Figures (3) to (6) for pinned ends, clamped-pinned ends, clamped ends and clamped-free ends respectively. Each figure describe the interaction relation between the end and intermediate axial loads represented by factors A_1 and A_2 for different locations of the intermediate load (α) and different moment of inertia ratio of lower segment to that of the upper one (n).



(a)



(*b*)



(*c*)

Fig. 3. Bucking loads parameters of columns with pinned ends (P-P) under intermediate load P_2 and end load P_1

(a) n=1.0, (b) n=2.0 and (c) n=4.0

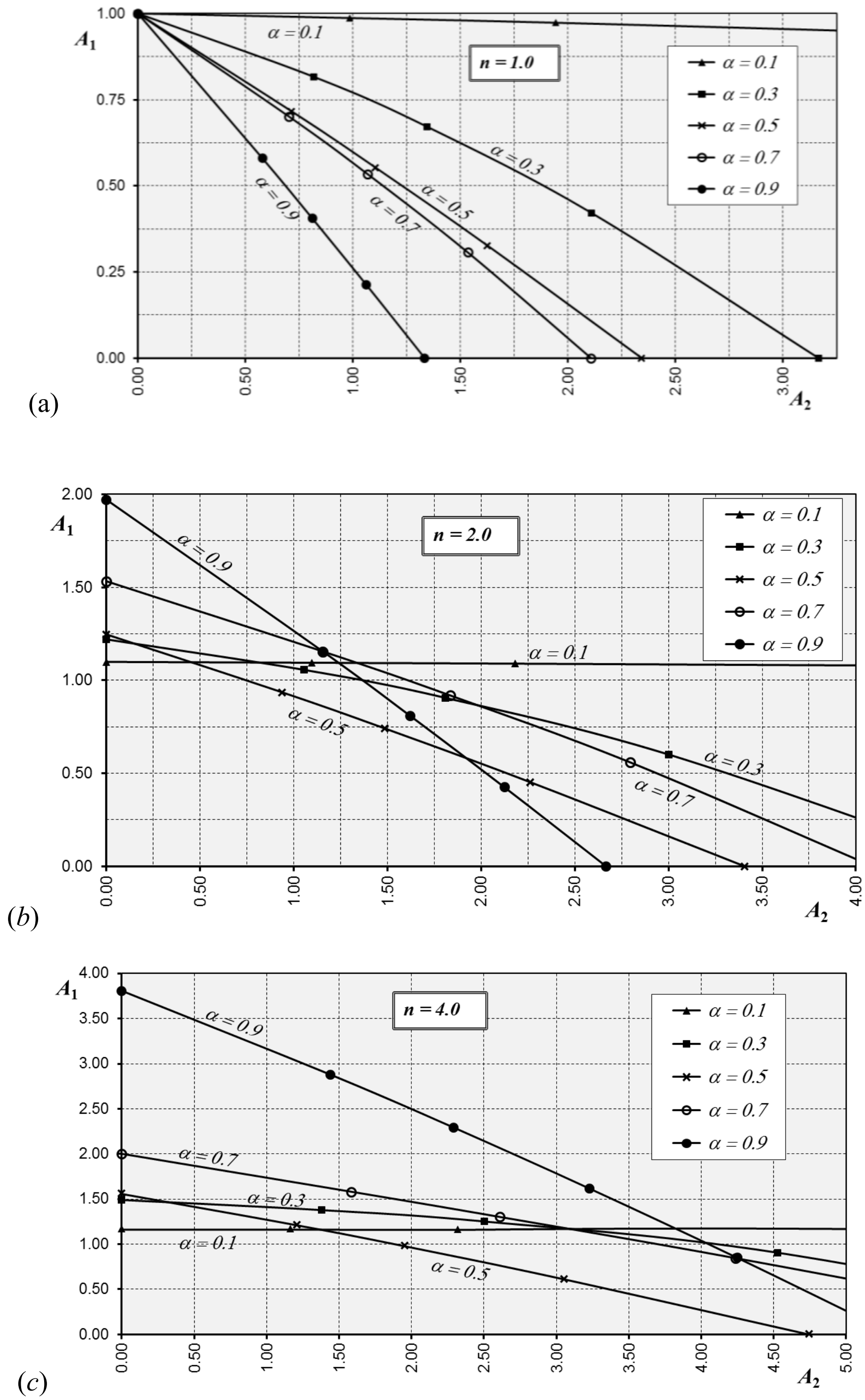


Fig. 4. Bucking loads parameters of columns with clamped – pinned ends (C-P).

under intermediate load P_2 and end load P_1

n=1.0, (b) n=2.0 and (c) n=4.0

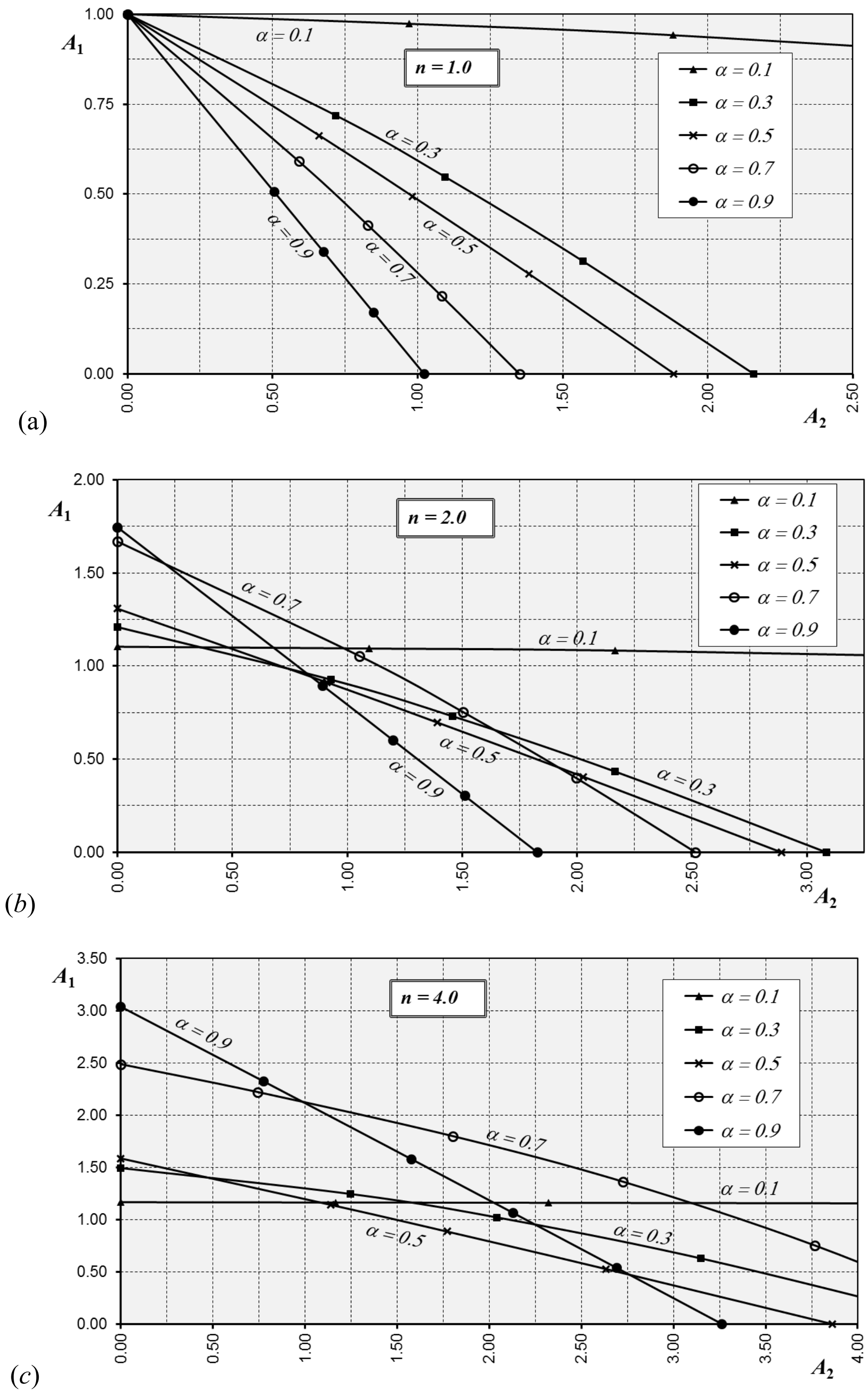


Fig. 5. Bucking loads parameters of columns with clamped ends (C-C) under intermediate load P_2 and end load P_1

(a) n=1.0, (b) n=2.0 and (c) n=4.0

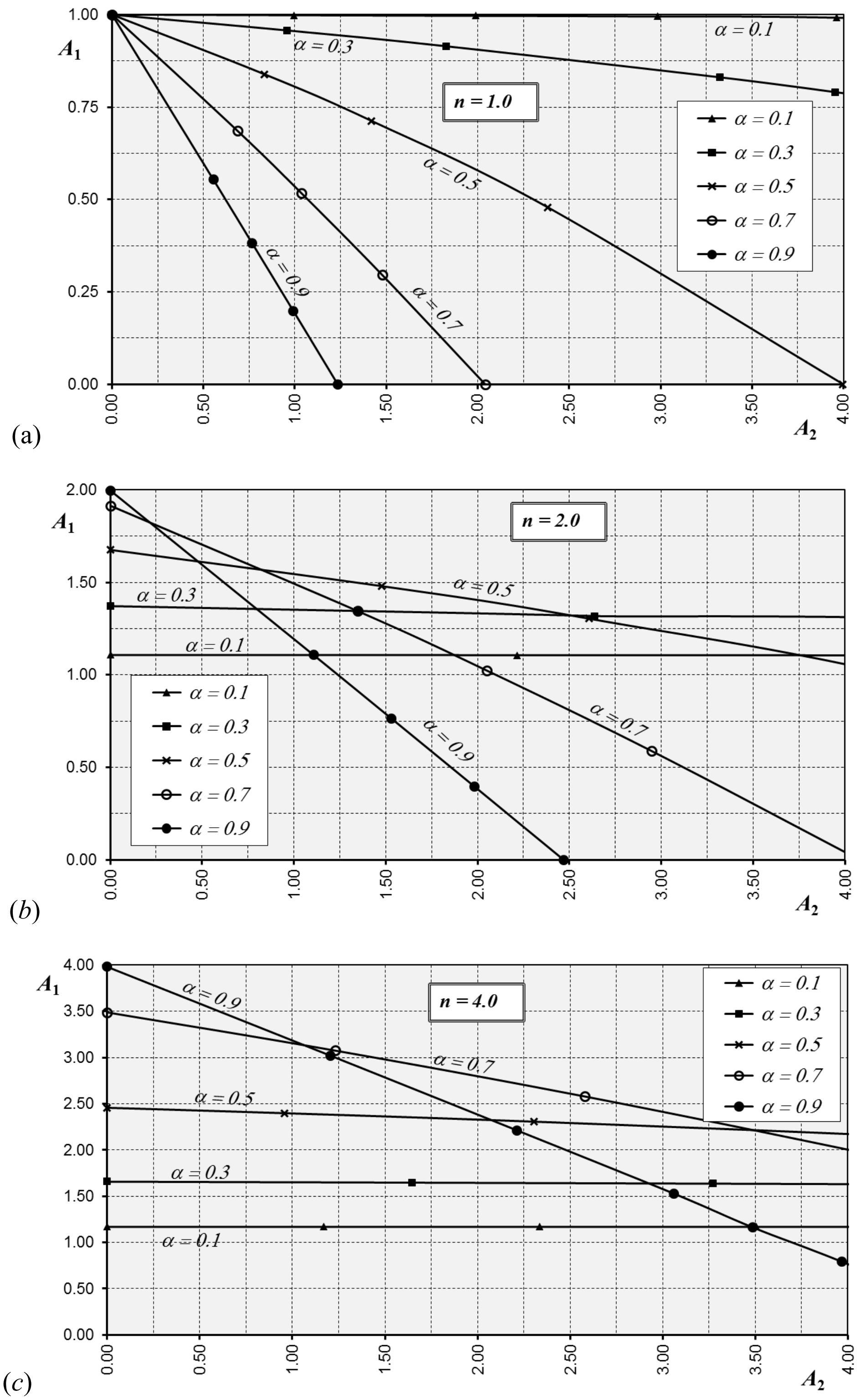


Fig. 6. Bucking loads parameters of columns with clamped - free ends (C-F) under intermediate load P_2 and end load P_1

(a) n=1.0, (b) n=2.0 and (c) n=4.0

From these figures, it is obvious that the buckling factor A_1 decrease almost linearly as the buckling factor A_2 increases. Curvature of some relations seems more pronounced for certain values of α different according the ratio n and the end conditions. Also, it can be noticed that when the intermediate axial load is absent (P_2=0), for different values of n, the buckling factor A_1 is very close to the exact buckling factor for each end conditions.

4. Comparison of the results

The obtained results are checked by comparison with the available results computed in the published references.

Table (1) shows the comparison of results obtained from this study with the exact results solved by Timoshenko and Gere [1] for the column with pinned ends considering the intermediate load location at the mid-span of the column (α =0.5).

Table 1
Comparison of the buckling load factor A_1 with exact results obtained by Timoshenko [1] for stepped P-P column (α =0.5).

m	*n*									
	1.00		1.25		1.50		1.75		2.00	
	Exact	F.E.M.	Exact	F.E.M.	Exact	F.E.M.	Exact	F.E.M.	Exact	F.E.M.
1.00	1.0000	1.0000	1.1077	1.1077	1.1883	1.1883	1.2500	1.2500	1.2985	1.2985
1.25	0.8882	0.8882	0.9900	0.9900	1.0671	1.0671	1.1268	1.1268	1.1739	1.1739
1.50	0.7981	0.7981	0.8939	0.8939	0.9673	0.9673	1.0246	1.0246	1.0702	1.0702
1.75	0.7240	0.7240	0.8142	0.8142	0.8840	0.8840	0.9388	0.9388	0.9826	0.9826
2.00	0.6623	0.6622	0.7472	0.7473	0.8135	0.8135	0.8658	0.8658	0.9079	0.9079
3.00	0.4926	0.4926	0.5609	0.5609	0.6153	0.6153	0.6589	0.6590	0.6945	0.6945

Another comparison with the exact results for uniform columns determined by C. M. Wang and I. M. Nazmul [2] is given in Table (2).

Also, a comparison with the results obtained by Pinarbasi and et.[5] using variational l iteration method (VIM) to solve the differential equations is given in Table (3).

These comparisons shows perfect match and the present method can be used simply by the designer engineers.

Table 2
Comparison of the buckling load factor A_1 with exact results obtained by Wang [2]
For uniform column (n=1.0).

BC	m	α 0.10		0.30		0.50		0.70		0.90	
		Exact	F.E.M.	Exact	F.E.M.	Exact	F.E.M.	Exact	F.E.M.	Exact	F.E.M.
C-F	1.33	0.99944	0.99945	0.98581	0.98581	0.94161	0.94162	0.86980	0.86980	0.78875	0.78876
	2	0.99835	0.99835	0.95726	0.95726	0.83782	0.83783	0.68578	0.68580	0.55388	0.55389
	4	0.99499	0.99499	0.87197	0.87198	0.61410	0.61412	0.41435	0.41435	0.29216	0.29216
P-P	1.33	0.93776	0.93778	0.86708	0.8671	0.85608	0.85611	0.84337	0.84338	0.78720	0.78721
	2	0.82743	0.82744	0.67982	0.67984	0.66224	0.66225	0.63764	0.6377	0.55070	0.55071
	4	0.59484	0.59485	0.40710	0.40711	0.39163	0.39164	0.36383	0.36383	0.28876	0.28876
C-P	1.33	0.99594	0.99595	0.93568	0.93569	0.88554	0.88556	0.87869	0.87870	0.80760	0.80760
	2	0.98743	0.98744	0.81696	0.81696	0.71611	0.71611	0.70178	0.70178	0.57969	0.57969
	4	0.95845	0.95845	0.56415	0.56416	0.44917	0.44917	0.42938	0.42939	0.31153	0.31153
C-C	1.33	0.99164	0.99164	0.89126	0.89127	0.85609	0.85609	0.81716	0.81716	0.75446	0.75446
	2	0.97330	0.97331	0.71846	0.71846	0.66220	0.66221	0.59088	0.59088	0.50580	0.50581
	4	0.90453	0.90453	0.43958	0.43958	0.39143	0.39143	0.31770	0.31770	0.25425	0.25425

Table 3.
Comparison of the buckling load factor A_1 (n=2.0) with VIM results [5].

BC	m	α 0.10		0.30		0.50		0.70		0.90	
		VIM	F.E.M.	VIM	F.E.M.	VIM	F.E.M.	VIM	F.E.M.	VIM	F.E.M.
C-F	1	1.10777	1.10789	1.37158	1.37172	1.67564	1.67578	1.91452	1.91465	1.99670	1.99676
	2	1.10723	1.10725	1.34554	1.34555	1.47949	1.47950	1.34554	1.34555	1.10723	1.10724
	4	1.10614	1.10626	1.28942	1.28955	1.15317	1.15327	0.82341	0.82347	0.58423	0.58425
P-P	1	1.00318	1.00318	1.07365	1.07365	1.29847	1.29847	1.70915	1.70915	1.98675	1.98675
	2	0.83493	0.83493	0.75824	0.75824	0.90788	0.90788	1.15679	1.1568	1.09903	1.09902
	4	0.60687	0.60688	0.46943	0.46943	0.56140	0.56141	0.68526	0.68526	0.57707	0.57707
C-P	1	1.09919	1.09920	1.22004	1.22004	1.24671	1.24671	1.53256	1.53256	1.97055	1.97055
	2	1.09455	1.09456	1.05608	1.05609	0.93607	0.93607	1.38487	1.15511	1.15494	1.15494
	4	1.08420	1.08421	0.78045	0.78045	0.61251	0.61251	0.75831	0.75831	0.62244	0.62244
C-C	1	1.10355	1.10356	1.20912	1.20912	1.30751	1.30751	1.66789	1.66789	1.74390	1.74390
	2	1.09393	1.09394	0.92726	0.92726	0.91170	0.91170	1.05339	1.05339	0.89361	0.89361
	4	1.06976	1.06976	0.59612	0.59612	0.56229	0.56230	0.58191	0.58191	0.45184	0.45184

5. Conclusions

Finite element method is performed to study the stability of two-segment stepped columns subjected to both intermediate and end axial loads. The classical end conditions are considered in this paper such as pinned ends, clamped ends, clamped-free ends and pinned-clamped ends.

Many curves that describe the interaction relation between the end and intermediate critical loads are introduced in this paper for each end conditions. These curves are given for various values of the intermediate load location ratio and the ratio between the flexural rigidity between the lower and upper parts. The obtained results can be obtained directly by design engineers and the desired method can be simply modelled by the designers.

The obtained results are compared with the available exact results for special cases and the other results in the published references and theses comparison show an excellent accuracy.

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