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Modal Analysis of a Thin-Walled Box-Girder Bridge and Railway Track Using Finite Element Framework

V. Verma [1*], A. Mallothu [2], K. Nallasivam [3]

1. Research Scholar, National Institute of Technology, Hamirpur, India
2. Master of Technology Student, National Institute of Technology, Hamirpur, India
3. Assistant Professor, National Institute of Technology, Hamirpur, India

Corresponding author: virajan75409@gmail.com

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ABSTRACT

Modal analysis has received widespread acceptance in past few decades for a wide range of applications. Bridges and buildings are two of the most popular structures that use this application in the context of civil engineering. The current study aims to apply finite element technique to estimate the free vibration characteristics of a railway track and a box-girder bridge. The curved bridge is numerically modeled using thin-walled box-beam finite elements that take into account torsional warping, distortion, and distortional warping, all of which are important characteristics of thin-walled box-girders. A commercially available finite element software ANSYS is used to simulate the railway track in three dimensions. The study is restricted to the initial design stage of a thin-walled box-girder bridge decks, in which a full three-dimensional finite element model is not required. For the thin-walled box-girder bridge, a MATLAB code has been built that yields the corresponding modal parameter results, whilst the modal parameters of the railway track system are acquired using ANSYS software.

1. Introduction

The use of curved alignment in bridges has become increasingly popular in recent years and hence box beam elements having exceptional bending and torsional rigidities are being used [1].

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The box-girder bridges offer a lot of advantages such as thinner sections, better serviceability and aesthetically pleasing designs. However, the complex structural actions which includes cross-sectional distortion and torsional warping present a huge challenge for their analysis and design [2].

Computer-aided modeling has also become quite popular among engineers for modeling and analysing various structures [3]. These structures may range from a tall building to a Railway track system. The Railway track system consists of Rails, Sleepers, Rail pads, Fastenings, Ballast and Sub-Ballast. Indian Railways forms an integral part of transportation within the country, which is the fourth largest network in the world. It therefore becomes of utmost importance to accurately model and analyse the railway track system so that heavy loads of railway traffic can be resisted [4]. An accurate mathematical model is preferred over an experimental investigation due to the heavy cost associated with the experimental practices. Box-girder bridges have been extensively used in many railway projects in India and hence the free vibration studies of these structures will form the stepping stone for understanding their dynamic behavior.

The free vibrational characteristics of any structure including modal parameters such as mode shapes and natural frequencies need to be studied to fully understand its fundamental dynamic behaviour [5]. Every structural system exhibits a specific pattern of vibration known as mode shape under a specific frequency called natural frequency. Different modes have different natural frequencies. Modes are a function of material properties like mass, stiffness, damping and boundary conditions of that structure. The bridge deck and railway track systems are some of the structural systems with continuous distribution of mass having infinite number of natural frequencies of which only very few lower frequencies are of practical significance.

Babu and Sujatha [6] implemented finite element approach for the analysis of railway track and calculated the track modulus for wooden and concrete sleepers. Sadeghi [7] carried out the free vibration studies of railway track by field investigation and obtained the mode shapes and frequencies for different conditions of the track system. Nguyen et al. [8] adopted various finite element models and studied the dynamic properties of railway loads on track system. Kaewunruen et al. [9] made use of the finite element software STRAND7 to predict the mode shapes and natural frequencies of a three-dimensional railway track system. The dynamic modeling of the railway track system was done by Kaewunruen and Remennikov [10] to study the free vibration behavior of concrete sleepers. Lu and McDowell [11] applied a discrete element model for railway ballast and studied various factors affecting its important mechanical properties. Shuber et al. [12] developed various theoretical models based on finite element techniques to check the dynamic response of railway track under harmonic loads. Garcia [13] did the finite element modeling of a railway track and examined the track safety for different combinations of velocity and displacement. Feng [14] studied the effect of various parameters on the dynamic behavior of railway track, where various vibration and static analysis was also performed. Le Pen [15] conducted studies on sleeper-ballast interaction and measured the response of railway track due to high-speed trains.

The past researchers have put in a good effort and used various methods for the free vibration characteristics of beams and bridges. Mukhopadhyay [16] presented an isoperimetric three-noded beam element to calculate vibration characteristics of horizontally curved beams using the finite element technique. The thin-walled beam theory of Vlasov [17] was effectively utilized by Noor et al. [18] for studying the free vibrations of curved thin-walled beams through simple finite element models. Panicker et al. [19] predicted the free vibration behaviour of FRP bridges by commercially available finite element software ABAQUS. Yoon et al. [20] provided the free vibration analysis of steel I-girder bridges by finite element formulation. The natural frequencies of horizontally curved beams were calculated by Snyder and Wilson [21] by applying a closed-form solution. Tabba and Turkstra [22] did a parametric study on the natural frequencies of thin-walled curved girders and gave a general solution for their free vibration behaviour. Memory et al. [23] found the mode shapes and natural frequencies of various bridges by applying Rayleigh's method. A dynamic model for the free vibration behaviour of thin-walled girder bridges was formulated by Kou et al. [24], which also included warping effects. Tan et al. [25] investigated the free vibration of the simply supported bridge by applying an analytic technique based on Euler-Bernoulli and transfer matrix method. Awall et al. [26] used some field measurements along with three dimensional finite element model to study the free vibration characteristics of an I-girder bridge. Yin et al. [27] utilised commercially available finite element software for predicting the modal parameters of a steel girder bridge. The free vibration analysis of composite I-girder bridges was conducted by Wodzinowski et al. [28] by performing various sensitivity analyses. Verma and Nallasivam [29,30] found the static response as well as factors affecting free vibration response of thin-walled box-girder bridge by adopting finite element formulation. The finite element methodology was extensively applied by Zhu et al. [31] for the vibrational study of thin walled rectangular beams. The flexural response of curved box-girder bridges was measured by Gupta and Kumar [32] using FEM software CSiBridge. Hamza et al. [33] presented a non-linear finite element model to estimate ultimate load capacity of horizontally curved steel beams. Tsiptsis and Sapountzaki [34][proposed a model in ABAQUS for the static and free vibrational analysis of straight and curved composite beams taking into account the friction model. Aggrawal et al. [35] performed some parametric analysis over box-girder bridges by varying both skew and curve angles and found that torsional moment ratio increased with the introduction of skewness. Tsiptsis and Sapountzakis [36] carried out an isogeometric analysis on the dynamic behavior of curved structures, taking into account the warping effects.

Research gap: It can be seen from the literature that various methods have been developed by previous researchers for the free vibration analysis of box-girders and railway track systems. However, none of these methods were fully able to justify the complex structural actions i.e. torsional and distortional warping of box-girders. The present approach not only takes into consideration these structural actions, but is equally effective in predicting its free vibration behaviour. Two different approaches of finite element technique have been utilised in the present work i.e. MATLAB coding has been done for box-girders and railway track system has been modeled using finite element software ANSYS.

Following are the objectives of the present study:

1. Finite element modelling of railway track system and thin-walled box-girder bridge.
2. Identifying the mode shapes and natural frequencies of the railway track system.
3. Predicting the free vibration characteristics of thin-walled box-girder with accuracy.

2. Assumptions and limitations

The basic assumptions related to the theory of linear elastic small displacement have been taken into consideration. Other assumptions and limitations used in this study include the following:

1. The Plane section remains plane, but may not remain perpendicular to the axis of the beam in case of bending and thus shear deformations are allowed.
2. The analysis is restricted to situations where the cross-section dimensions are relatively small compared to the length of the span and the radius of the curvature.
3. The stiffness of Diaphragms is assumed to be infinite in their plane, whereas they are considered to be perfectly flexible perpendicular to the plane.
4. The study is limited to the case where the thickness of the wall is small compared to the cross-section dimensions.
5. The study limits for the preliminary design phase of box-girder bridge decks, where a complete three-dimensional finite element analysis is unnecessary.
6. The material for rail and sleeper is steel and concrete respectively, both homogeneous and isotropic and the Gross compressive pressure of concrete is measured as 0.003.
7. Rails are assumed to have the same behavior in both compression and tension.
8. There is a perfect bond between materials, which have no self-weight.

3. Methodology adopted for thin-walled box-girder

3.1. Element geometry

A curved thin-walled box beam element has been shown in Fig. 1, where the cross-section is created using straight lines. The distortion effect can be studied by assuming that the axis of symmetry of the cross-section is vertical, whereas the analysis of torsion and bending does-not require this assumption.

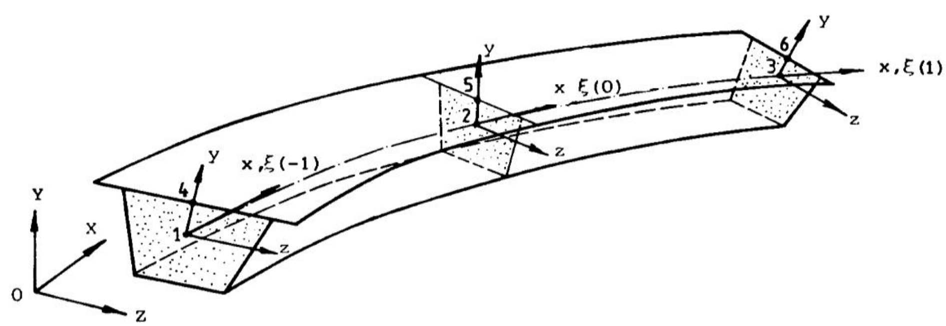

Fig. 1. Three noded thin-walled box beam element.

The element axis is defined as the locus of the centroids which may be eccentric but parallel to the flexural axis. The three elemental nodes that are located on the axis are at the ends and at the mid-point.

The element is expressed by a local Cartesian coordinate system (x, y, z), which is directed towards the axis curve. The centroid of the cross-section denotes the origin of the coordinate system. It is assumed that the principal axis of the cross-section coincides with the direction of local yz axis. The local x axis is in the direction of the elemental axis and the tangent from node one to node three. The vertical axis of symmetry is denoted by the local y axis and a right-handed orthogonal system is used to describe the local z axis.

The Global coordinate system is expressed in the form of a natural coordinate ξ. The value of ξ is -1, 0 and 1 on the three faces of the element. Let $r = X.i + Y.j + Z.k$ indicates the position vector of any point 'P' on the elemental axis, then a unit tangent vector directed towards the x direction is given by

$$e_x = J^{-1}\left(\frac{\partial X}{\partial \xi} i + \frac{\partial Y}{\partial \xi} j + \frac{\partial Z}{\partial \xi} k\right) \tag{1}$$

The unit vectors in the direction of global X, Y and Z are represented by i, j and k in the above equation. The Jacobian factor is defined as

$$J = \left[\left(\frac{\partial X}{\partial \xi}\right)^2 + \left(\frac{\partial Y}{\partial \xi}\right)^2 + \left(\frac{\partial Z}{\partial \xi}\right)^2\right]^{1/2} \tag{2}$$

The unit tangent vector towards the local z direction is a cross product of the terms given by

$$e_z = e_x \times e_y \tag{3}$$

3.2. Relationship between stress and strain

The displacements are defined in local as well as global coordinate system. The displacements in the local coordinate system are written as,

$$\bar{\delta} = \left[u, v, w, \theta_x, \theta_y, \theta_z, \theta_x', \gamma_d, \gamma_d'\right]^T \tag{4}$$

In the above equation, the translations in the local x, y and z axes are represented by u, v and w respectively, the twisting angle by θ_x, the twisting rate by θ_x', rotation around y and z axes by θ_y and θ_z respectively, the angle of distortion by γ_d and the distortion rate by γ_d'.

The above displacements are also defined in the global coordinate system as,

$$\delta = \left[U, V, W, \varphi_x, \varphi_y, \varphi_z, \theta_x', \gamma_d, \gamma_d'\right] \tag{5}$$

In the above equation, the translations in the global x, y and z axes are represented by U, V and W respectively and the rotations around the same axes by φ_x, φ_y and φ_z respectively. The twisting rate θ_x', the angle of distortion γ_d and the distortion rate γ_d' remains in local coordinate system as earlier. It is therefore evident that there are nine degrees of freedom per node of the thin-walled box beam element.

The stress vector in its general form is given by,

$$\sigma = \left[N_x, Q_y, Q_z, M_{st}, M_y, M_z, \frac{1}{\mu_t} B_1, M_d, B_{11}\right]^T \tag{6}$$

In the above equation, the axial force is represented by N_x, shear forces by Q_y and Q_z, pure torsional moment by M_{st}, primary bending moments by M_y and M_z, warping shear parameter by μ_t, torsional warping bi-moment by B_1, distortional moment by M_d and distortional warping bi-moment by B_{11}.

The strain vector in its general form is given by,

$$\varepsilon = \left[\varepsilon_x, \varepsilon_{yx}, \varepsilon_{zx}, \psi_{\theta x}, \psi_{yx}, \psi_{zx}, \psi_{wtx}, \psi_{dx}, \psi_{wdx}\right]^T \tag{7}$$

The different parameters in the above equation are defined as follows:

Axial strain, $\varepsilon_x = \frac{\partial u}{\partial x}$ (8)

Shear strain in y-direction, $\varepsilon_{yx} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} - \theta_z$ (9)

Shear strain in z-direction, $\varepsilon_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = \frac{\partial w}{\partial x} + \theta_y$ (10)

Torsional strain, $\psi_{\theta x} = \frac{\partial \theta_x}{\partial x}$ (11)

Flexural strain about y-axis, $\psi_{yx} = \frac{\partial^2 u}{\partial x \partial z} = \frac{\partial \theta_y}{\partial x}$ (12)

Flexural strain about z-axis, $\psi_{zx} = \frac{\partial^2 u}{\partial x \partial y} = -\frac{\partial \theta_z}{\partial x}$ (13)

Torsional warping strain, $\psi_{wtx} = -\frac{\partial^2 \theta_x}{\partial x^2} - \frac{1}{R}\frac{\partial \theta_z}{\partial x}$ (14)

$'R'$ in the above equation represents the radius of curvature and the torsional warping strain has been modified to consider the initial curvature effect.

Distortional strain, $\psi_{dx} = \gamma_d$ (15)

Distortional warping strain, $\psi_{wdx} = -\frac{\partial^2 \gamma_d}{\partial x^2}$ (16)

The rigidity matrix in its generalized form is

$$D=\begin{bmatrix} EA & & & & & & & & \\ & GA_{sy} & & & & 0 & & & \\ & & GA_{sz} & & & & & & \\ & & & GJ_T & & & & & \\ & & & & EI_y & & & & \\ & & & & & EI_z & & & \\ & & 0 & & & & \frac{1}{\mu_t}E_1J_I & & \\ & & & & & & & E_1J_D & \\ & & & & & & & & E_1J_{II} \end{bmatrix} \tag{17}$$

In the above equation, Young's modulus of Elasticity is represented by E, shear modulus by G, the cross-sectional area by A, effective shear areas along y and z directions by A_{sy} and A_{sz} respectively, the torsional moment of inertia by J_T, the primary bending moments of inertia around y and z axes by I_y and I_z respectively, the torsional warping moment of inertia by J_I, warping shear parameter by μ_t, distortional second moment of area by J_d and the distortional warping moment of inertia by J_{II}. E_1 represents the conversion modulus of elasticity and is defined as,

$$E_1 = \frac{E}{(1-\nu^2)} \tag{18}$$

Where, ν is the Poisson's ratio.

3.3. Eigen value problem for un-damped system

The general equation of motion for an un-damped free vibration system can be written as

$$[M][\ddot{\delta}] + [K][\delta] = 0 \tag{19}$$

$[K]$ and $[M]$ are the global stiffness and mass matrix after applying boundary conditions.

Assuming harmonic motion in natural mode of vibration, the response can be written as

$$\{\delta\} = \{X\}sin(\omega t + \varphi) \tag{20}$$

where $\{X\}$ is the vector of nodal amplitude of vibration, ω is the circular natural frequency of vibration [rad/sec] and φ is the phase angle.

Substitution of Eq. (37) in Eq. (36) leads to the generalized eigen value problem.

$$\{[K] - \omega^2[M]\}\{X\} = 0 \tag{21}$$

Eq. (21) is solved using a standard eigen solver to obtain the values of natural frequencies and mode shapes of the box-girder bridge.

4. Methodology adopted for railway track

4.1. Mathematical model

The Beam on elastic foundation (BOEF) model has made tremendous improvement in the railway track design and forms the backbone of the current study. Fig. 2 depicts a typical BOEF system. The model was presented by Winkler in 1867 and is still being used to find the deflection of the track.

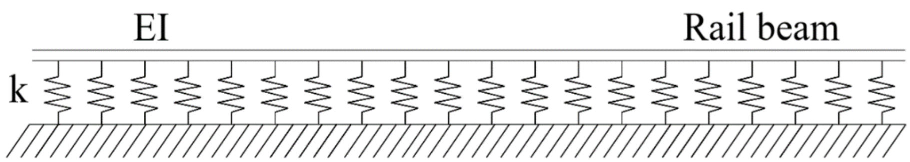

Fig. 2. Beam on elastic foundation system.

The model is quite simple with a clear physics and exceptional mathematical formulation. The rails are depicted as an infinite length Euler-Bernoulli (E-B) beam having continuous supporting Winkler foundation base beneath them. The Winkler foundation is defined as an infinite longitudinal line of uncoupled, elastic springs. The deflection of the beam is proportional to the distributed force through which the beam is supported. The rail deflection w(x) is defined by the equation given as

$$EI\,\frac{\mathrm{d}^4\mathrm{w(x)}}{dx^4} + kw(x) = q(x) \qquad (22)$$

where, x is the length coordinate, $q(x)$ is the load on the rail, EI is the bending stiffness of beam (Nm^2) and k is the foundation stiffness (N/m^2)

This model gives satisfactory results when the track subjected to a static load rests on soft support, such as wooden sleepers. The assumptions and limitations of the BOEF model include assuming a continuous foundation and response to be linear, not accounting for shear deformations and considering the material behavior in only vertical direction. Also, the tensile stress developed between the sleeper and rest of the foundation is different from the compressive stresses which causes an unusual lifting of track and both front and back wheel. This phenomenon gives improper results in a granular model and hence a tensionless railway track model is desirable, which has an indefinite contact zone and depends on the load.

5. Types of analysis

Three different types of analysis are mainly done after the modelling process. This study however focusses on linear eigenvalue analysis for the prediction of mode shapes and natural frequencies. The total length of the track taken for the model is 100 meters and a standard distance of 1.676 meters i.e. broad gauge is taken for the rails. Table 1 lists the material properties of various components of the track system used in this study.

Table 1
Properties of different components of railway track.

Material	Density (kN/m^3)	Young's Modulus	Poisson's Ratio
Ballast	186.4	$21*10^9$	0.3
Rail	7650	$210*10^9$	0.33
Sleeper	765.0	$2100*10^9$	0.2
Formation layer	160	$5*10^9$	0.4
Sub-Ballast 1	158.5	$5*10^9$	0.4
Sub-Ballast 2	159.5	$5.5*10^9$	0.4
Sub-Ballast 3	160.5	$6*10^9$	0.4
Sub-Grade 1	175.7	$0.0435*10^9$	0.38
Sub-Grade 2	180.7	$0.0535*10^9$	0.38
Sub-Grade 3	185.7	$0.0635*10^9$	0.38

5.1. Linear eigenvalue analysis

Linear Eigenvalue study is conducted for the estimation of natural frequencies and mode shapes of a vibrating system. The eigenvalue of a vibrating system can be calculated by two different methods i.e. Lanczos and Subspace algorithms. Lanczos eigen technique is used in cases where a large number of eigenvalues are required. The limit set determines the range of subspace eigen solver, however any range can be chosen in Lanczos.

5.2. General static analysis

The response of structures subjected to static loading is done by general static analysis which takes into account both linear and non-linear effects. The load application differs from the dynamic case and the load is increased in small fractions in this analysis. Various Nonlinear effects i.e. material nonlinearities, displacement and friction can be seen and a single point load is applied to the railway track model.

5.3. Dynamic implicit analysis

The dynamic implicit method is used to measure the transient dynamic behavior of a structure under a time-varying load, where the loading depends on the analysis being carried out. A smaller increment in time is preferred in dynamic analysis, which depends on the variation in structure. Two different time increments are used for this analysis i.e. Automatic and fixed. The automatic increment is applied for the general implicit dynamic integration method, whereas half-step control relates to the accurate dynamic solution.

6. Modeling procedures in ANSYS

The procedure followed in the design of the track system and various component properties has been shown here. The modeling techniques which have been used in ANSYS to design the railway track system have also been addressed. The result is a three dimensional railway track model based on finite elements created using ANSYS-APDL. ANSYS is considered to be one of the most efficient and flexible software for modeling various structures.

The Railway track system consisting of rail, sleeper, ballast and subgrade has been modeled with spring damper systems, which takes into consideration the effects on the performance of the real track system. The analysis based on the finite element technique is governed by assumed strains and degree of freedom at each node. The unknowns are found by forming a matrix, which represents the accurate equation at each node.

6.1. Elements type

6.1.1. SOLID 185

The rails, which have an I section are modelled using SOLID 185 element as seen in Fig. 3.

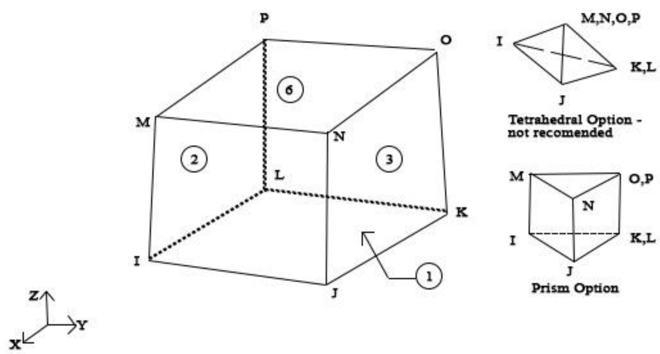

Fig. 3. SOLID 185 element.

6.1.2. SOLID 65

The sleepers are modelled using SOLID 65 element, which is shown in Fig. 4.

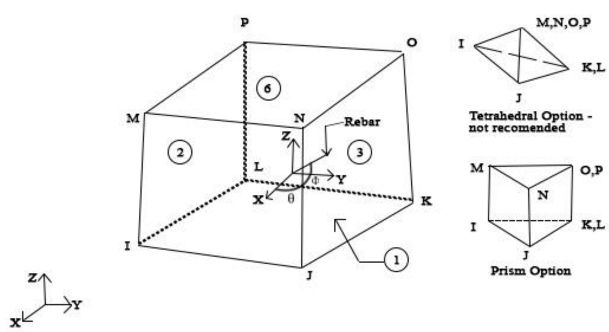

Fig. 4. SOLID 65 element.

6.2. Modeling

The modelling in ANSYS is done by creating areas and volumes with an extensive use of springs and dashpots. The bed of the railway track is modelled by creating areas and stacking different layers, whereas the ballast is modelled as a rigid mass. Fig. 5 demonstrates a complete three dimensional model of the railway track superstructure and substructure.

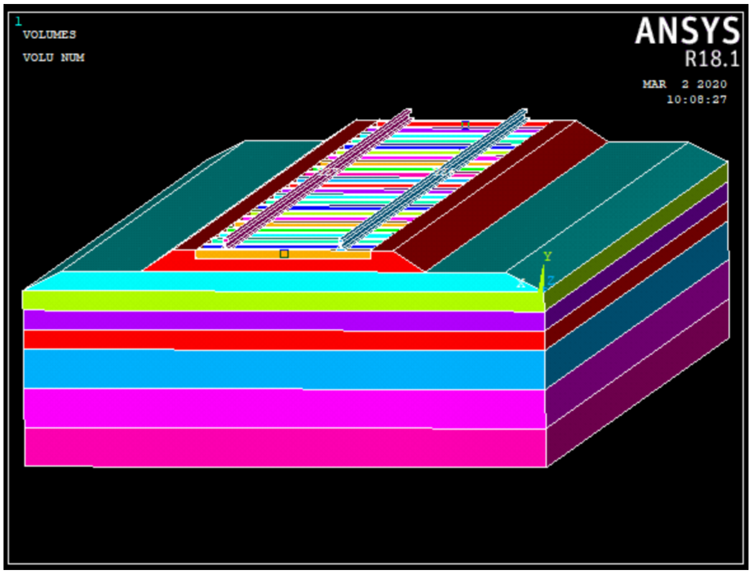

Fig. 5. Railway track model.

6.3. Meshing

Meshing is an integrated component of a simulation model, wherein complex geometries are segregated and are used for a discrete local approximation of a greater area. The speed and accuracy of the simulation are greatly improved by the meshing process. The track bed is modelled using a rectangular or square mesh. Fig. 6 shows the rail track model after mesh generation. The meshing of the volumes involves the following steps:

1. Select the volume option in the main menu bar and select the desired element.
2. Select mesh attributes in the meshing tool option and the element defined in the previous step.
3. Lastly, the mesh model option in the mesh tools is selected to assign the desired attributes to the selected element.

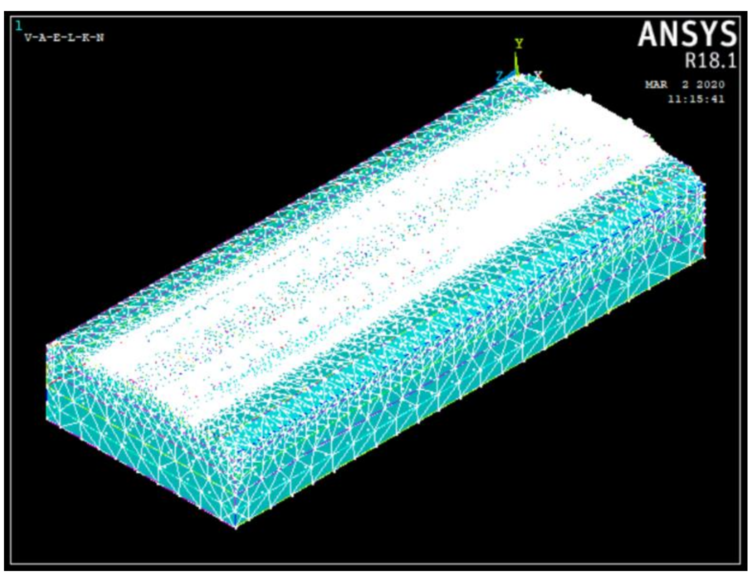

Fig. 6. Rail Track Model after mesh generation in ANSYS.

7. Results and discussions

7.1. Simply supported straight box-girder

This numerical example taken from Kermani and Waldron [2] considers a simply supported straight box-girder having 30 meters span, wherein the ends are braced by diaphragms. Table 2 shows various sectional properties of the girder and Fig. 7 depicts the cross-section of the girder. A total of 30 thin-walled box-girder elements are used to analyse the beam. Fig. 8(a)-(j) shows the mode shape diagrams for the first 10 modes, whereas various cyclic frequencies are shown in Table 3. The values of cyclic frequency increase with each mode, which is self-explanatory.

Table 2
Different sectional properties.

Sectional Property	Value
E	3.45 e+10 N/m^2
ν	0.15
G	1.5e+10N/m^2
I_z	1.6061 m^4
J_T	3.2593 m^4
J_I	0.79304 m^6
J_{II}	1.01304 m^6
J_d	0.00563 m^2
μ_t	0.365
A	3.7050 m^2

Table 3
Cyclic frequencies for first ten modes.

Mode	Cyclic Frequency (Hz)	Cyclic Frequency (Hz)
1	v (1stVertical Mode)	4.3312
2	w (1stLateral Mode)	12.6911
3	v (2ndVertical Mode)	16.8054
4	γ_d(1stDistortional Mode)	23.8779
5	γ_d(2ndDistortional Mode)	26.9586
6	u (1stAxial Mode)	31.5726
7	v (3rdVertical Mode)	36.3955
8	γ_d(3rdDistortional Mode)	37.5648
9	θ_x(1stTorsional Mode)	38.9932
10	w (2ndLateral Mode)	44.5813

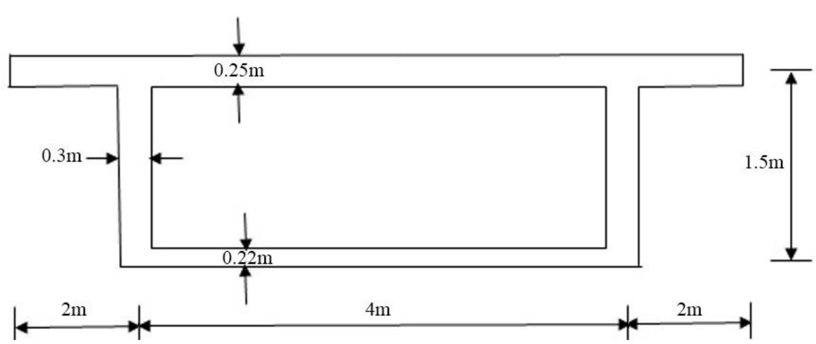

Fig. 7. Straight beam model with dimensions.

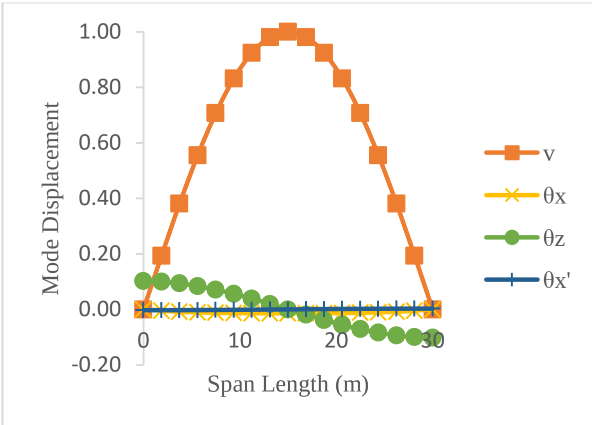

Fig. 8(a) 1st Vertical Mode.

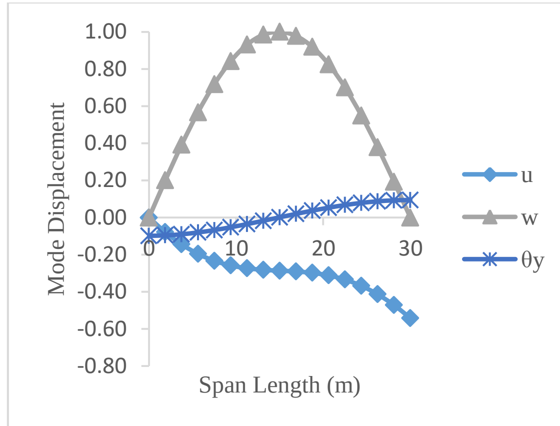

Fig. 8(b) 1st Lateral Mode.

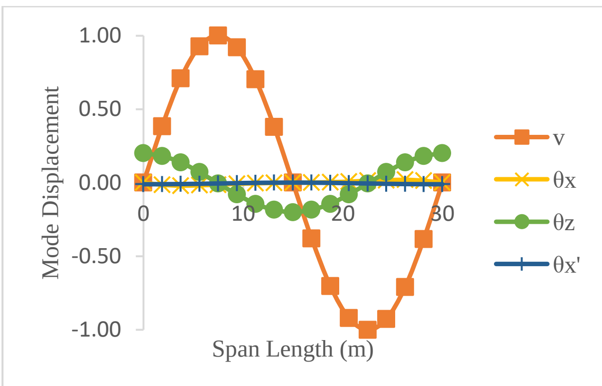

Fig. 8(c) 2nd Vertical Mode.

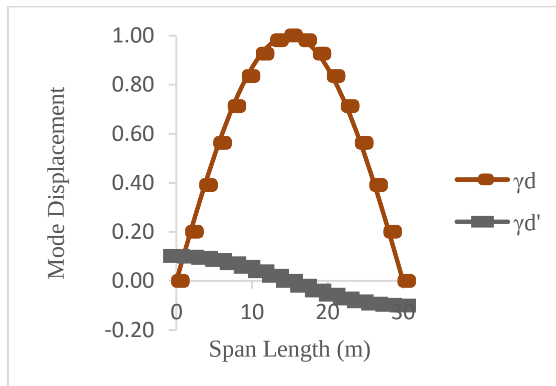

Fig. 8(d) 1st Distortional Mode.

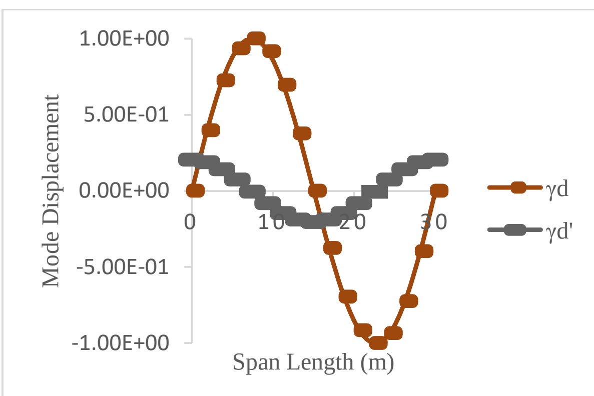

Fig. 8(e) 2nd Distortional Mode.

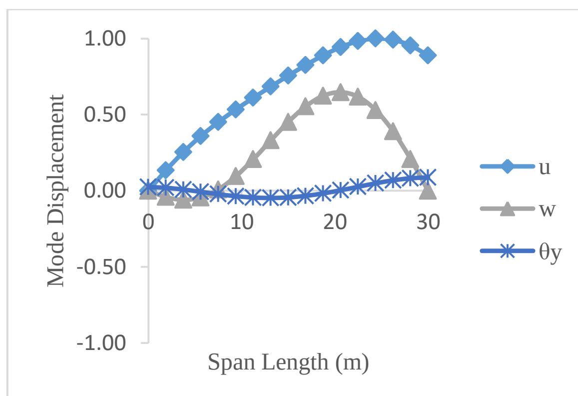

Fig. 8(f) 1st Axial Mode.

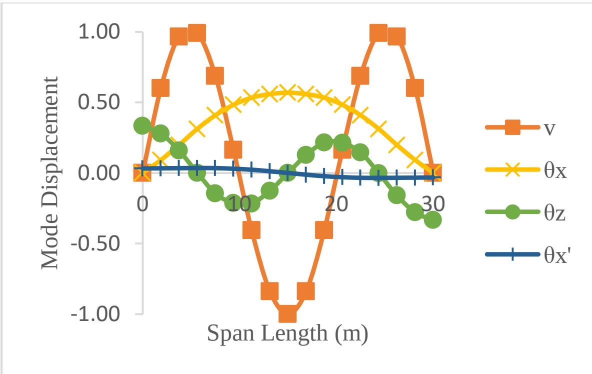

Fig. 8(g) 3rdVertical Mode.

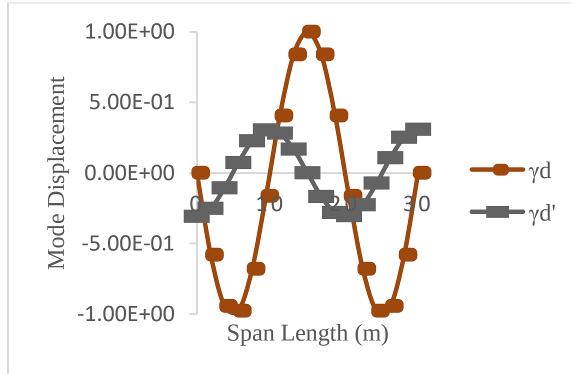

Fig. 8(h) 3rdDistortional Mode.

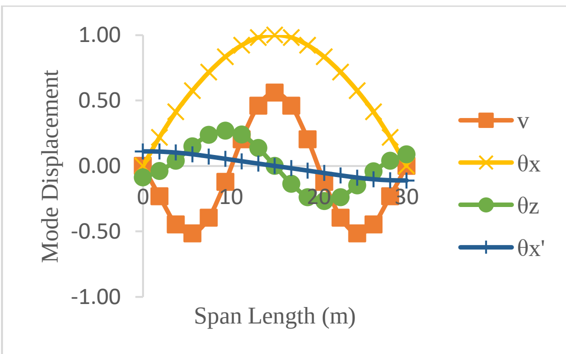

Fig. 8(i) 1stTorsional Mode.

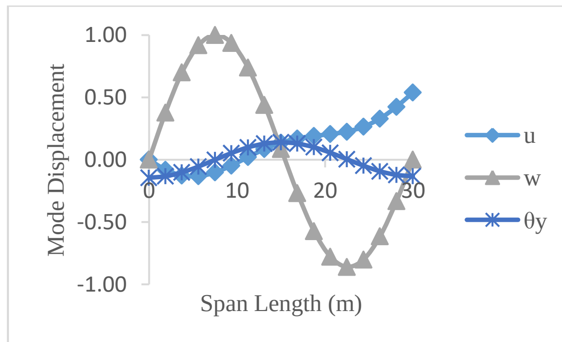

Fig. 8(j) 2ndLateral Mode.

7.2. Railway track system

The modal analysis results of track system have been presented in ANSYS software, which includes mode shape diagrams for the first ten natural frequencies. The simulation results shows the various modes of the rail as a whole of the structure is analysed and as the frequency starts increasing, the lateral vibration of the track can be seen. There is an advantage in doing free vibration modal analysis of the track system as the modal analysis analyzes the whole structure and global modes of rail can be seen. The track structure vibrates when the frequency starts from zero. The vibration of the track is first seen in the lateral direction, after vibration of rails as simply supported beam. The various mode shapes are illustrated in Fig. 9(a)-(j) and frequencies are listed in Table 4.

Table 4
Cyclic frequencies for first ten modes

Mode	Cyclic Frequency (Hz)	Cyclic Frequency (Hz)
1	First mode	261.49
2	Second mode	261.49
3	Third mode	261.50
4	Fourth mode	309.30
5	Fifth mode	309.31
6	Sixth mode	309.31
7	Seventh mode	396.40
8	Eighth mode	396.40
9	Ninth mode	396.40
10	Tenth mode	489.42

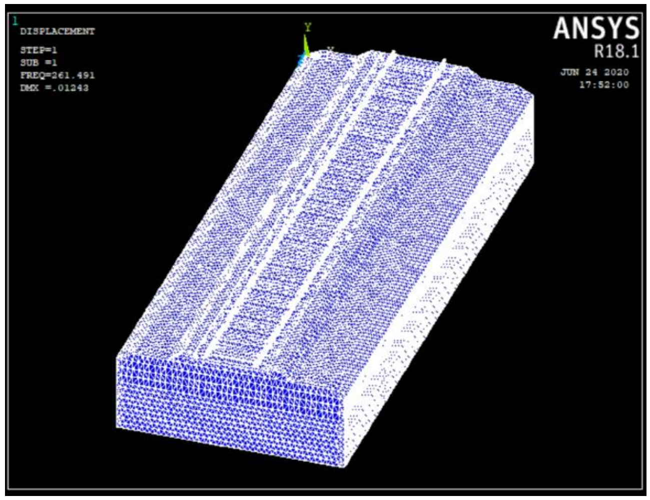

Fig. 9(a) First mode.

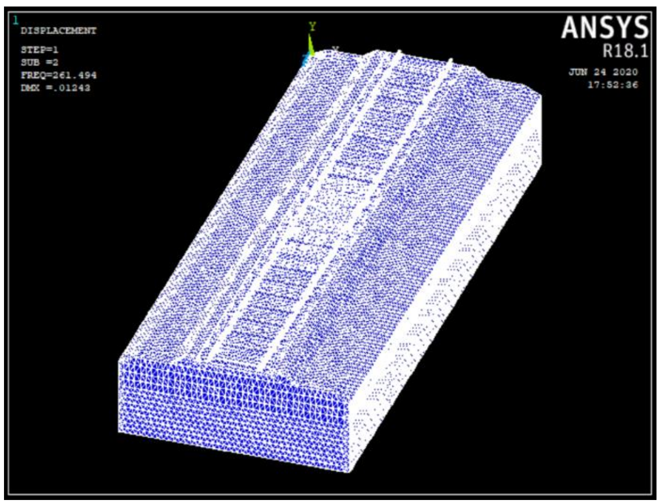

Fig. 9(b) Second mode.

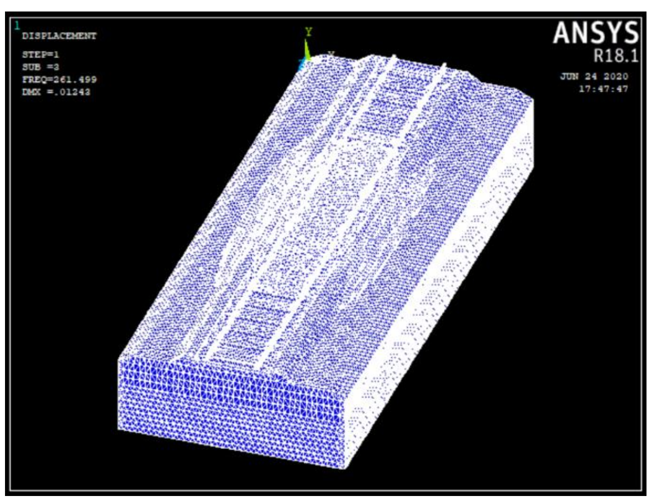

Fig. 9(c) Third mode.

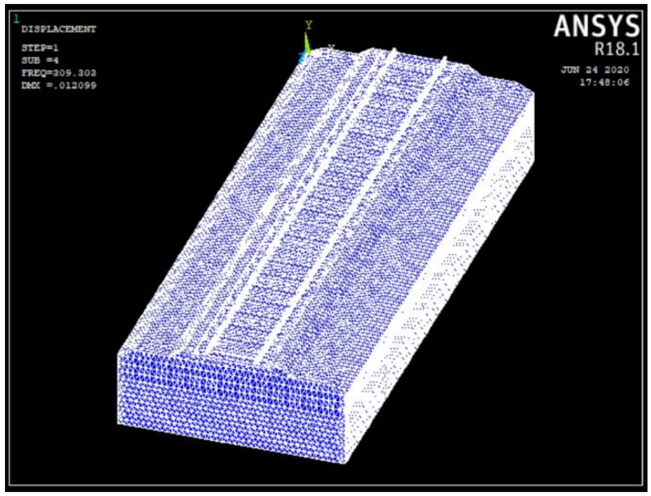

Fig. 9(d) Fourth mode.

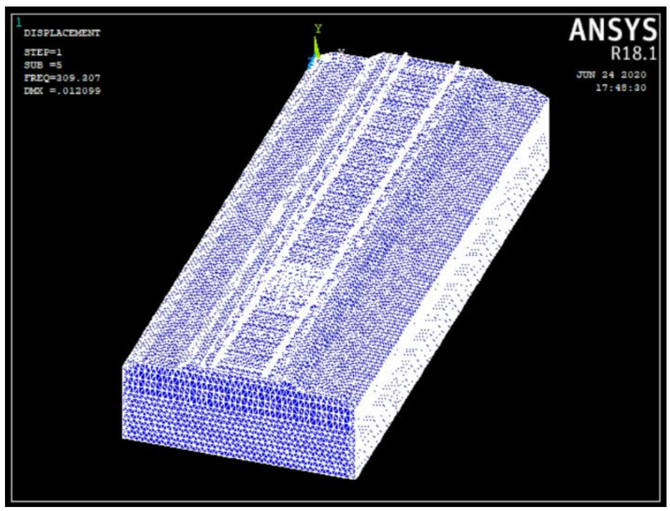

Fig. 9(e) Fifth mode.

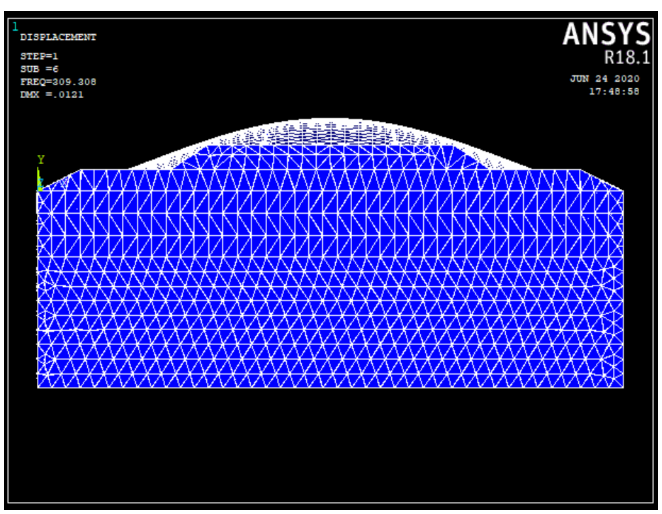

Fig. 9(f) Sixth mode.

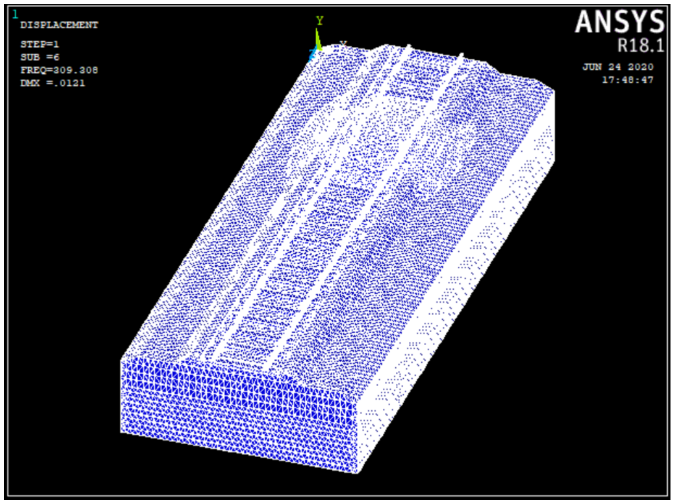

Fig. 9(g) Seventh mode.

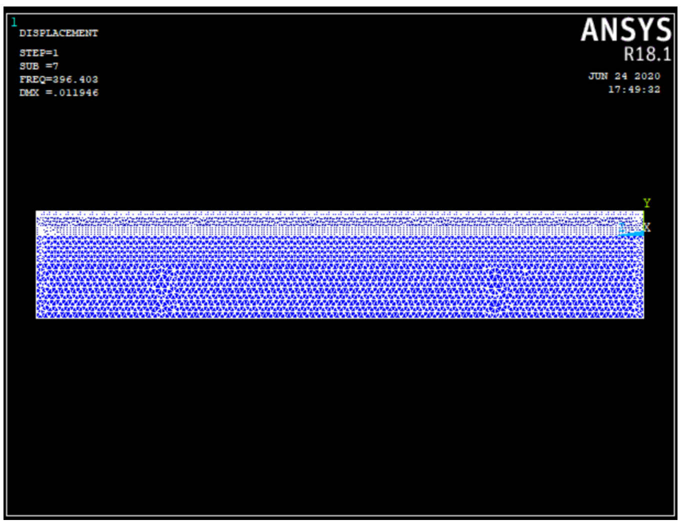

Fig. 9(h) Eighth mode.

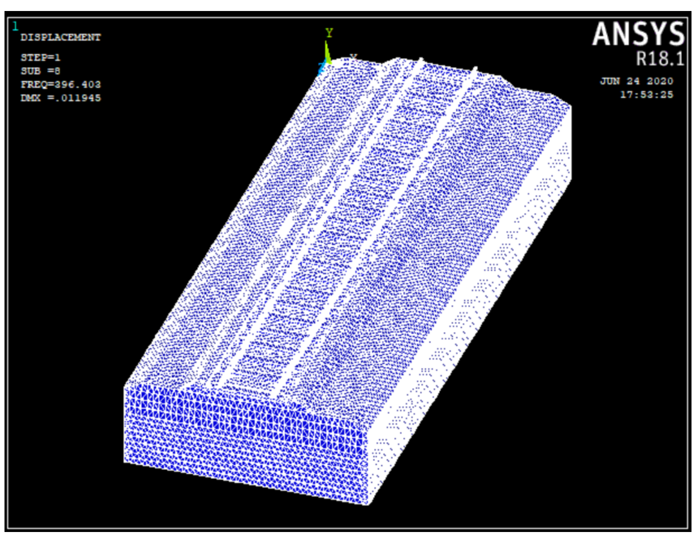

Fig. 9(i) Ninth mode.

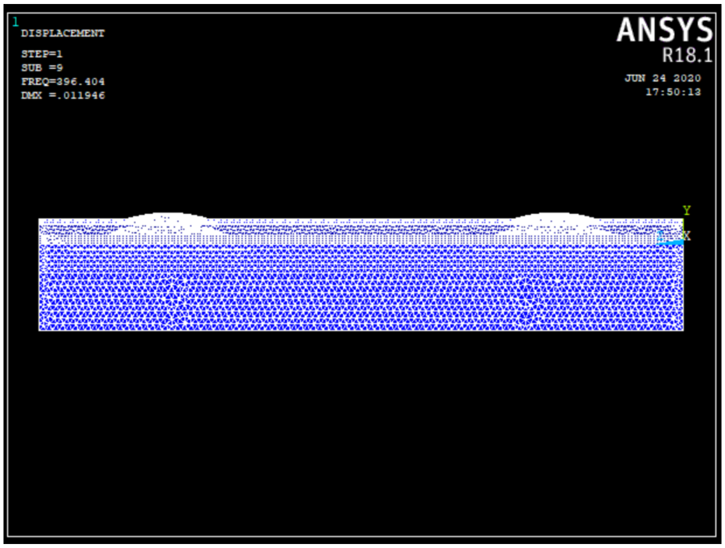

Fig. 9(j) Tenth mode.

8. Conclusion

The railway track is modelled using Solid 65 and 185 element and the box-girder bridge has been modelled using a three-noded thin-walled box-beam element. The pertinence of such an element for dynamic analysis has been confirmed by testing the modal parameters of the curved box girder model. Following points conclude the key aspects of the work presented in this paper:

1. The finite element methodology adopted gives a reasonable representation of all the complicated structural actions of a thin-walled box-girder and hence can be effectively utilized for conducting its modal analysis studies.
2. The modal analysis results of the bridge showed that the vertical, lateral and distortional modes dominated and the first cyclic frequency was recorded as 4.33 Hz. The maximum frequency value of 44.58 Hz was for the second lateral mode.
3. The free vibration of the railway track is first seen in the lateral direction after vibration of rails as simply supported beam. The first cyclic frequency value was measured as 261.49 Hz and the highest value was seen as 489.42 Hz.

Conflict of interest

The authors state that there is no conflict of interests.

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