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Modelling of the Compressive Strength of Palm-Nut-Fibre Concrete Using Scheffe's Theory

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ABSTRACT

In this research study, a mathematical model is developed to optimize the palm-nut-fiber reinforced concrete's compressive strength using Scheffe's (5, 2) simplex-lattice design. Palm-nut-fiber which is an agricultural residue obtained after the processing of palm-oil is utilized as the fifth component in concrete consisting of water, cement, fine and coarse aggregates. Fibers are used to help fresh concrete to keep it from cracking and plastic shrinkage and also for a concrete structure of complicated or complex geometry where the use of the conventional rebar will not work. The compressive strength of Palm-nut-fiber were obtained for the different componential ratios using Scheffe's Simplex method and for the control points which will be utilized for the validation of the Scheffe's model. The model's adequacy was tested using student's t-test and ANOVA at 5% critical value. The statistical result indicates a good relationship between the values obtained from the developed Scheffe's model and the control laboratory results. The maximum value of compressive strength of the palm-nut fiber concrete obtained was 31.53Nmm^2 corresponding to mix ratio of 0.525:1.0:1.45:1.75:0.6 and minimum value of compressive strength obtained was found to be 17.25Nmm^2 corresponding to mix ratio of 0.6:1.0:1.8:2.5:1.2. For water, Limestone Portland cement (LPC), fine aggregate, coarse aggregate and palm nut fiber respectively. Using the developed Scheffe's simplex model, the proportion of the mixture ingredients to a certain prescribed compressive strength value can be estimated with a high degree of accuracy and also providing the solution in less amount of time.

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1. Introduction

Concrete require material with ductility capacity due to the fact that there are many sources of stresses which the concrete on its own cannot resist due to its brittle capacity. In most cases, steel reinforcement are utilized in order to enhance the concrete's strength performance. Addition of reinforcement in concrete changes the failure type of the concrete from brittle failure to ductile failure observe the formation of failure cracks before there exists tremendous loss of strength; this induces plastic deformation capacity after yielding of the material under stress. By so doing, the prospect of total collapse of the structure without early warning signs will be properly taken care of while maintenance culture will be embraced to avert the crises from happening [1].

Sometimes we hope to make a concrete structure of complicated or complex geometry where the use of the conventional rebar will not work; in this case the use of fiber reinforced concrete is highly recommended. Fiber are like micro reinforcement and when the fiber reinforced concrete beam is loaded, the beam fall apart under failure load, but it will be held together by thin fibers. Fibers not only make the concrete stronger but also makes it hold the load longer after failure limit is reached. If we look at the cracks developed at the fiber reinforced concrete beam, we can see the fibers doing an important job to give the concrete post-cracking strength also known as toughness [2,3].

Utilization of fiber in concrete mixes to obtain a fiber-reinforced concrete is seriously gaining much popularity as a viable alternative to enhancement of concrete durability and strength performance. Fiber concrete is specifically utilized for the construction of complex or irregular geometry structures such as tunneling, loading decks, concrete pads, bridge decks, thin unbounded layers and concrete slabs. The major role of fibers in concrete is to alter its cracking mechanism, this structural behavior modification causes the macro cracking to turn into micro cracking. This reduction in the cracking failure mechanism will eventually improve the permeability property of the cracked concrete while also enhances its ultimate cracking strain property [4].

The brittle unreinforced concrete breaks completely once there is an occurrence of failure crack, this means that the unreinforced concrete has a very low load carrying ability at failure limit of the material. This can be solved by the introduction of fiber in concrete especially for complex geometry; fiber reinforced concrete is able to carry sustainable amount of load across the crack or failure region. Fiber reinforced concrete does not fail totally after the occurrence of the initial crack, the fibers arrange in a matrix form at distinct locations and is able to absorb loads at failure region unlike the unreinforced concrete material [5].

Empirical method of concrete mix design consists of series of extensive tests majorly centered on the bases of trial and error which involves rough estimates based on practical experience without a theoretical or statistical methodological approach. In order to limit the number of trial and error tests before obtaining of optimal result respect to the response parameter, developing an analytical methods which will rationalize the initial trial mix into a systematic and logical process. This will help in locating the optimum combination for the mixture ingredients in consuming less resources based on laid down knowledge of certain empirical relationships, specific weights of mixture ingredients and results from past literatures [6,7].

Scheffe's method is a mixture model technique utilized for the adjustment of statistical significance levels to account for multiple comparison in a linear regression analysis. It is very essential for a special case of regression analysis termed analysis of variance, when performing evaluation of simultaneous confidence levels for regression analysis involving objective functions. The aim of this research study is to assess the addition of palm-nut fiber in a concrete consisting of cement, water, fine aggregate and coarse aggregates to obtain a five component concrete mix, to evaluate the applicability of Scheffe's simplex lattice optimization theory to ascertain the optimal mixture combination in the palm-nut fiber reinforced concrete in terms of compressive strength properties and to develop mathematical model for the optimization of the response parameters [8,9].

The use of Scheffe's simplex lattice design to achieve mixture design have been applied in several civil engineering applications to proffer solutions in such areas as soil stabilization, geotechnical material science, pavement materials modifications and concrete technology [10]. Okere *et al*, [11]; in their work on the flexural strength of soilcrete blocks made with laterite and the optimization using Scheffe's simplex lattice design method. Laboratory tests were carried out with respect to the calculated Scheffe's design point. Statistical analysis were carried out to validate the developed mathematical model. The maximum response value of the 1.452N/mm^2 was obtained. From the research results, lateritic soil which is readily available and affordable has been used successfully to produce soilcrete blocks.

Alaneme *et al*, [12]; in their research study on the utilization of Scheffe's theory for the optimization of the flexural strength property of the palm-nut fiber concrete. The concrete mixture have five components namely; cement, water, coarse aggregates, fine aggregates and an agricultural waste known as palm-nut fiber. The optimal combination of 0.525:1.0:1.45:1.75:0.6 was obtained at a strength value of 11.40 N/mm^2 while the minimum combination ratio of 0.6:1.0:2.0:2.8:1.1 for water, cement, fine and coarse aggregate and palm nut fiber respectively was obtained at a strength vale of 5.35 N/mm^2.

Chijioke Chiemela *et al,* [13]; in their work the use of Scheffe's theory to model the compressive strength property of concrete when provided with componential ratios while also predicting the corresponding portions of the mixture ingredients with prescribed value of compressive strength vale of concrete obtained from substitution of the conventional river sand as the fine aggregates by quarry dust. The developed model was further tested for adequacy with the control point's response value. This statistical analysis method used are f-statistics and student's t-test at 95 % confidence level. The result shows that there is no significant difference between the predicted and measured values.

2. Methodology

2.1. Mathematical modelling and mix proportion formulation

Scheffe formulated a model for the assessment of the response of a particular characteristics of a mixture with respect to some variations in the proportions of its component materials [14]. In his simplex lattice model, he considered experiments whereby the desired response obtained is relatively proportional to the ratio of ingredients combination. A mixture experiments is utilized in a scenario whereby the independent variables which are the ingredients' combination ratio is actually not independent but they are interrelated by some of series of imposed constraints which enable a homogenous mixture to be obtained [15].

Lattice is a properly arranged settings of space with points that are uniformly distributed in a simplex; a $\{q, m\}$ simplex is a structural representation of the intersecting hyper-planes between the experimental points of the mixture; q represents the total number of mixture components while m represents the order of the regression polynomial [16]. The factor space takes a form of a regular $(q-1)$ simplex due to the imposed sum to one constraint on the mixture design; this is presented in eqn. 1 below

$$\sum_{i=1}^{q} x_i = 1 \qquad (1)$$

$x_i \geq 0$ for concentration of the components in the mixture and q represents the total number of mixture components. The points division along the simplex by each component on a straight line takes m+1 values equally spaced from each other ranging from point 0 to 1; it is mathematically represented in eqn.2 below

$$x_i = 0, \frac{1}{m}, \frac{2}{m}, \ldots, 1 \text{ for } i = 1, 2, \ldots, q. \qquad (2)$$

Scheffe observed mixtures experiments whereby the response parameter depends on the ratio of the components' combination and not on the quantity of the mixture [8]. The sought for parameter or property of interest is presented using equation of a polynomial form as shown in eqn. 3 below

$$y = b_o + \sum b_i x_j + \sum b_{ij} x_i x_j + \sum b_{ijk} x_i x_j x_k + \ldots + e \qquad (3)$$

Where $b_o, b_i, b_{ij}, b_{ijk}$ are constants, x_i, x_j, x_k represents the pseudo components and for second order polynomial the canonical order is expressed in eqn. 4 below

$$y = b_o + \sum b_i x_i + \sum b_{ij} x_i x_j + e \qquad (4)$$

Further expansion of Equation (4) by substituting $(0 \leq i \leq j \leq 5)$ into the values of i and j transforms to

$$Y = b_o + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11}X_1^2 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{15}X_1X_5 + b_{22}X_2^2 + b_{23} X_2X_3 + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{33}X_3^2 + b_{34}X_3X_4 + b_{35}X_3X_5 + b_{44}X_4^2 + b_{45}X_4X_5 + b_{55}X_5^2 \qquad (5)$$

Multiplying eqn. (1) by b_o (5)

$$b_0 = b_0 (X_1 + X_2 + X_3 + X_4 + X_5) \qquad (6)$$

Multiplying in succession Eqn. (1) by X_1, X_2, X_3, X_4, and X_5 and Substituting Equations (6) into Equation (4) we obtained the general second order polynomial model form for five component mixture. This is presented in eqn. 7

$$\hat{Y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \beta_{45}X_4X_5 \quad (7)$$

thus eqn. 7 could be presented as follows;

$$y = \sum \beta_i x_i + \sum \beta_{ij} x_i x_j \text{ Where } i \geq 1 \text{and } i \leq j \leq 5 \quad (8)$$

Where X_i represents the pseudo components for the mixture design while β_i represents the response coefficients of Scheffe's optimization equation. This coefficients can be expressed as β_i which is for the pure or binary blends and as β_{ij} which is for the ternary blends or the combination of the mixture components. They can be defined as follows;

$$\beta_i = Y_i \text{ and } \beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j$$

Thus

$$\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2,\ \beta_{13} = 4Y_{13} - 2Y_1 - 2Y_3,\ \beta_{14} = 4Y_{14} - 2Y_1 - 2Y_4,\ \beta_{15} = 4Y_{15} - 2Y_1 - 2Y_5,$$

$$\beta_{23} = 4Y_{23} - 2Y_2 - 2Y_3,\ \beta_{24} = 4Y_{24} - 2Y_2 - 2Y_4,\ \beta_{25} = 4Y_{25} - 2Y_2 - 2Y_5,\ \beta_{34} = 4Y_{34} - 2Y_3 - 2Y_4,$$

$$\beta_{35} = 4Y_{35} - 2Y_3 - 2Y_5,\ \beta_{45} = 4Y_{45} - 2Y_4 - 2Y_5 \quad (9)$$

Eqn. 9 shows the relationship between Scheffe's regression coefficients and the actual response

2.2. Number of coefficients

The number of coefficient for the Scheffe's five component mixture can be computed using eqn. (10). This number also moderates the number of run for the experiment and also for the control points too.

$$n = \frac{(P + M - 1)!}{M!(P - 1)!} \quad (10)$$

Where P which is the total amount of mixture components is 5 and M which is the polynomial order is 2

$$N = \frac{(5 + 2 - 1)!}{2!\,(5 - 1)!} = N = \frac{6!}{2!\,4!} = 15$$

2.3. Five component factor space

The points on the vertices of the factor space represent pure or binary component blends which indicates hundred percent mixture of a single mixture component. The pure or binary blend are assigned at the vertex of the simplex factor space. For the five component mixture, we have five vertices and ten spread in between the vertices of the simplex. All mixture interior to the perimeter of the simplex region are blends of all of the q-components. The factor space is the space within which all the experimental points will be distributed [17].

The first five pseudo component for the {5, 2} simplex represents the position of the binary blend of the mixture which are located at the vertices of the tetrahedron simplex.

A1 [1:0:0:0:0], **A2** [0:1:0:0:0], **A3** [0:0:1:0:0], **A4** [0:0:0:1:0], **A5** [0:0:0:0:1].

While the next ten other pseudo mix ratios remaining which are located at mid points of the lines joining the vertices of the simplex is presented below

A12 [0.5:0.5:0:0:0], **A13** [0.5:0:0.5:0:0], **A14** [0.5:0:0:0.5:0], **A15** [0.5:0:0:0:0.5], **A23** [0:0.5:05:0:0], **A24** [05:0:0:0.5:0], **A25** [0:0.5:0:0:0.5], **A34** [0:0:0.5:05:0], **A35** [0:0:0.5:0:0.5], **A45** [0:0:0:0.5:0.5].

The actual and pseudo components are related with a mathematical expression in eqn. 11

$$Z = AX \tag{11}$$

Where Z represents the concentration of the actual components, X represents the respective value for the pseudo components and A which is $n \times n$ matrix where n is equal to the total number of mixture ingredients; for this design, we obtain a five by five matrix which is obtained from the first five run of the mixture ratios. These mix ratios are shown in eqn. 12;

$\mathbf{Z_1}$ [0.45:1.0:1.25:1.45:0.2], $\mathbf{Z_2}$ [0.5:1.0:1.35:1.6:0.4], $\mathbf{Z_3}$ [0.55:1.0:1.55:1.9:0.8], $\mathbf{Z_4}$ [0.6:1.0:1.8:2.5:1.2], $\mathbf{Z_5}$ [0.65:1.0:2.0:3.0:1.8].

Substitution of X_i and Z_i into Equation (8) using the corresponding pseudo components to determine the corresponding actual mixture components.

$\mathbf{Z_1}$ = water cement ratio; $\mathbf{Z_2}$ = cement; $\mathbf{Z_3}$= fine aggregate; $\mathbf{Z_4}$ = coarse aggregate; $\mathbf{Z_5}$ = palm nut fiber

Substituting the obtained initial five run of mixes, we have the [A] matrix

$$\begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix}$$

The [A] matrix is further used to calculate the real proportion [Z] by applying eqn. (8); substituting the respective values of the pseudo components to obtain the matrix table shown in Table 1 and Table 2 for the control points.

3. Experimental program

3.1. Materials

The materials assessed for this research study are mixture of coarse and aggregate, cement, water, and palm nut fiber. The cement used is Dangote Limestone Portland cement (LPC) conforming to British Standard Institution BS 12 (1978). For the fine aggregate material used, the grain size ranges from 0.05 - 4.5 mm which was gotten from the local river. The type of water used for the mixture experiment is a borehole clean water. For the Coarse aggregate the grain size ranges from 12.5 mm to 4.75 mm and is sourced from a stone market. Also for the agricultural waste palm nut fiber, it was sourced from a palm oil mill at Oboro in Ikwuano L.G.A, Abia state.

3.2. Compressive strength test

The test specimens used for the compressive strength experiments were concrete cubes. They were cast in steel mould measuring 150mm*150mm*150mm. The mould and its base were damped together during concrete casting to prevent leakage of mortar. Engine oil was spread with a soft brush across the inner surface of the moulds to ensure easy removal of the set concrete cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted water cement ratios and mix ratios. A number of 30 mix proportions were used to produce 90 concrete cube which implies three replicates for each experimental point. Fifteen (15) out of the 30 mix ratios will be for the control points which will be used to produce 45 cubes for the conformation of the adequacy of the developed mathematical model for the optimization of compressive strength of palm nut fiber reinforced concrete. Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28days of curing the specimens were taken out of the curing tank and compressive strength determined. Three concrete cubes will be cast for each mixture and cured at 28 days in which the average compressive strength will be determined after crushing [18,19].

The compressive strength was then calculated using the formula below:

$$\text{Compressive strength} = \frac{\text{average failure load}(\text{N})}{\text{cross-sectional area}(\text{mm}^2)} = \frac{P}{A} \tag{12}$$

3.3. Durability performance determination

The durability performance of palm nut fiber reinforced concrete sample soaked in a salty water for 14 days was evaluated through compressive strength by weight loss and reduction in strength. The responses gotten were compared to the control tests which are concrete cured under normal condition to ascertain the durability performance of the palm nut fiber concrete using the formula below;

$$\frac{f'_{abs} - f'_s}{f'_{abs}} \times 100\% \tag{13}$$

f'_{abs} represents the mean strength of the concrete specimen in normal condition (control) and f'_s is the mean strength for concrete soaked for 14 days.

For the weight loss relationship, it is presented in eqn. 14 below

$$\frac{W_{abs} - W_s}{W_{abs}} \times 100\% \tag{14}$$

W_{abs} represents the mean weight of the concrete specimen in normal condition (control) and W_s is the mean weight for concrete soaked for 14 days.

4. Results and discussion

4.1. Physical and Chemical Properties of Test Materials.

The physical and chemical properties test results of the mixture ingredients namely were presented so as to observe the general engineering behavior of the test materials which is very influential to the response parameters.

4.1.1. Chemical properties of palm-nut

The physical and chemical properties of the fiber used for the experiment is presented in table 1 below; the results obtained indicated that the agricultural waste called palm-nut fiber which will be used for the mixture experiments possesses high carbon and oxygen content and very low percentages of sulfur, magnesium and silicon.

Table 1
Chemical Properties of Palm-nut Fiber.

Elements	N	H	C	Mg	Ca	O	Na	Si	Cl	S
Atomic percentage (%)	0.32	1.85	54.88	0.35	1.78	38.22	0.69	0.78	0.85	0.28

Parameters	Moisture content (%)	Ash (%)	higher heating value (MJ/Kg)	Specific gravity	fiber size (mm)	Water absorption (%)
Values	37.5	6.23	19.44	1.24	12 - 20	21.8

4.1.2. Physical properties of aggregates

The aggregates physical properties with respect to water absorption, specific gravity, fineness modulus and particle size distribution analysis results were used to evaluate the physical properties of the aggregates. The results are presented in table 2 below; from the results, the coarse aggregate produced a fineness modulus and specific gravity result of 6.88 and 2.68 respectively while the fine aggregates produced 2.79 and 2.62 respectively.

Table 2
Physical properties of Aggregate Materials.

Physical and Mechanical Properties	Coarse Aggregate	Fine Aggregates
Specific gravity	2.68	2.62
Water absorption (%)	0.22	-
Fineness Modulus	6.88	2.79

4.1.3. The particle size distribution of aggregates in the concrete mix

The grain size distribution test results for the coarse and fine aggregates which is plotted on a semi-log graph through a cumulative frequency curve so as to obtain its gradation parameters.

The result is presented in fig.1 below; From the sieve analysis results, for the coarse aggregates, 98.86 % and 2.07 % finer was obtained for 12.7 mm and 1.18 mm sieve sizes respectively; while for the fine aggregates, 99.47 % and 1.02 % finer was obtained for 4.75 mm and 0.075 mm sieve sizes respectively.

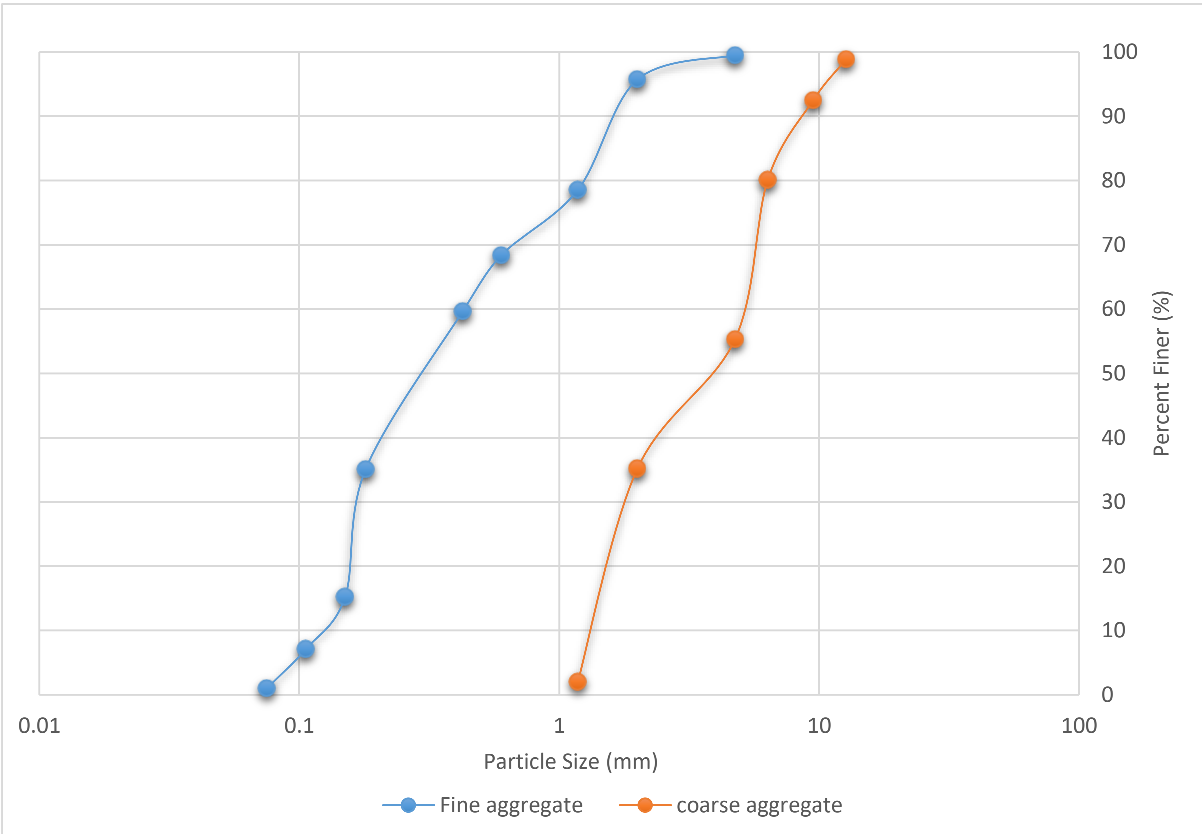


Fig 1. Grain Size Distribution of Aggregate Materials.

Using British standard soil classification system detailed in BS 5930. For the coarse aggregates, it comprises of 19.87 % of medium gravel, 44.92 % of fine gravel and 33.14 % of coarse sand while for the fine aggregates, it comprises of 4.28 % of fine gravel, 27.31 % of coarse sand, 8.77 % of medium sand and 58.62 % of fine sand.

4.2. Mixture design component ratio formulation

The mixture ingredients ratio formulation design is achieved using Scheffe's simplex-lattice design approach of {5, 2} simplex; possessing five components and at second order regression polynomial. From the design, fifteen experimental points which are required to satisfy the condition of m+1 points for intersecting q components in a simplex factor space are utilized. Fifteen extra points will also be designed which represents the control points which are used for the validation of the generated Scheffe's model. The mixture design is initiated by setting the mixture points at the vertices of the simplex; since we have five vertices representing the binary or pure blends. These points represent the first five points of the design fifteen Scheffe's coefficients with their corresponding pseudo components in binary vales. The remaining ten points represents the ternary points of the simplex factor space and are obtained by utilizing the mathematical relationship between the actual and pseudo component ratios expressed in equation 11 above. The mathematical equation is also used to calculate the actual ratio values for the control points and these mixture proportion are then taken to the laboratory to generate their respective responses in terms of compressive strength value. These mixture proportions are presented in tables 3 and 4 below.

Table 3

Matrix Table for Scheffe's {5, 2} - Lattice Polynomial for Compressive Strength Test.

ACTUAL						PSEUDO				
Z1	**Z2**	**Z3**	**Z4**	**Z5**	**RESPONSE**	**X1**	**X2**	**X3**	**X4**	**X5**
0.45	1	1.25	1.45	0.2	Y1	1	0	0	0	0
0.5	1	1.35	1.6	0.4	Y2	0	1	0	0	0
0.55	1	1.55	1.9	0.8	Y3	0	0	1	0	0
0.6	1	1.8	2.5	1.2	Y4	0	0	0	1	0
0.65	1	2	3	1.8	Y5	0	0	0	0	1
0.475	1	1.3	1.525	0.3	Y12	0.5	0.5	0	0	0
0.5	1	1.4	1.675	0.5	Y13	0.5	0	0.5	0	0
0.525	1	1.525	1.975	0.7	Y14	0.5	0	0	0.5	0
0.55	1	1.625	2.225	1	Y15	0.5	0	0	0	0.5
0.525	1	1.45	1.75	0.6	Y23	0	0.5	0.5	0	0
0.55	1	1.575	2.05	0.8	Y24	0	0.5	0	0.5	0
0.575	1	1.675	2.3	1.1	Y25	0	0.5	0	0	0.5
0.575	1	1.675	2.2	1	Y34	0	0	0.5	0.5	0
0.6	1	1.775	2.45	1.3	Y35	0	0	0.5	0	0.5
0.625	1	1.9	2.75	1.5	Y45	0	0	0	0.5	0.5

4.3. Compressive strength (response, Yi)

The laboratory response values in terms of compressive strength of the concrete cube samples cured for 28 days was obtained for the fifteen different mixture design points with a total of three replicates for each design point. The values are utilized for the development of the Scheffe's model for the optimization of the compressive strength property of palm-nut fiber concrete. The results are presented in fig. 2 below; from the results, we observe that points corresponding to Y_1 and Y_{23} generated the maximum value at 30.29 MPa and 31.53 MPa respectively while Y_4 and Y_{45} generated the minimum values at 17.25 MPa and 18.30 MPa respectively. These results indicates a better performance in terms of the strength property of the concrete due to the introduction of fiber matrix.

Table 4
Mixture Proportion of Control Points for Compressive Strength Test.

ACTUAL						PSEUDO				
Z1	**Z2**	**Z3**	**Z4**	**Z5**	**RESPONSE**	**X1**	**X2**	**X3**	**X4**	**X5**
0.525	1	1.4875	1.8625	0.65	C1	0.25	0.25	0.25	0.25	0
0.5375	1	1.5375	1.9875	0.8	C2	0.25	0.25	0.25	0	0.25
0.55	1	1.6	2.1375	0.9	C3	0.25	0.25	0	0.25	0.25
0.5625	1	1.65	2.2125	1	C4	0.25	0	0.25	0.25	0.25
0.575	1	1.675	2.25	1.05	C5	0	0.25	0.25	0.25	0.25
0.55	1	1.59	2.09	0.88	C12	0.2	0.2	0.2	0.2	0.2
0.51	1	1.425	1.735	0.54	C13	0.3	0.3	0.3	0.1	0
0.515	1	1.445	1.785	0.6	C14	0.3	0.3	0.3	0	0.1
0.53	1	1.52	1.965	0.72	C15	0.3	0.3	0	0.3	0.1
0.545	1	1.58	2.055	0.84	C23	0.3	0	0.3	0.3	0.1
0.56	1	1.61	2.1	0.9	C24	0	0.3	0.3	0.3	0.1
0.585	1	1.73	2.365	1.16	C25	0.1	0	0.3	0.3	0.3
0.57	1	1.67	2.275	1.04	C34	0.1	0.3	0	0.3	0.3
0.555	1	1.595	2.095	0.92	C35	0.1	0.3	0.3	0	0.3
0.54	1	1.535	1.945	0.74	C45	0.1	0.3	0.3	0.3	0

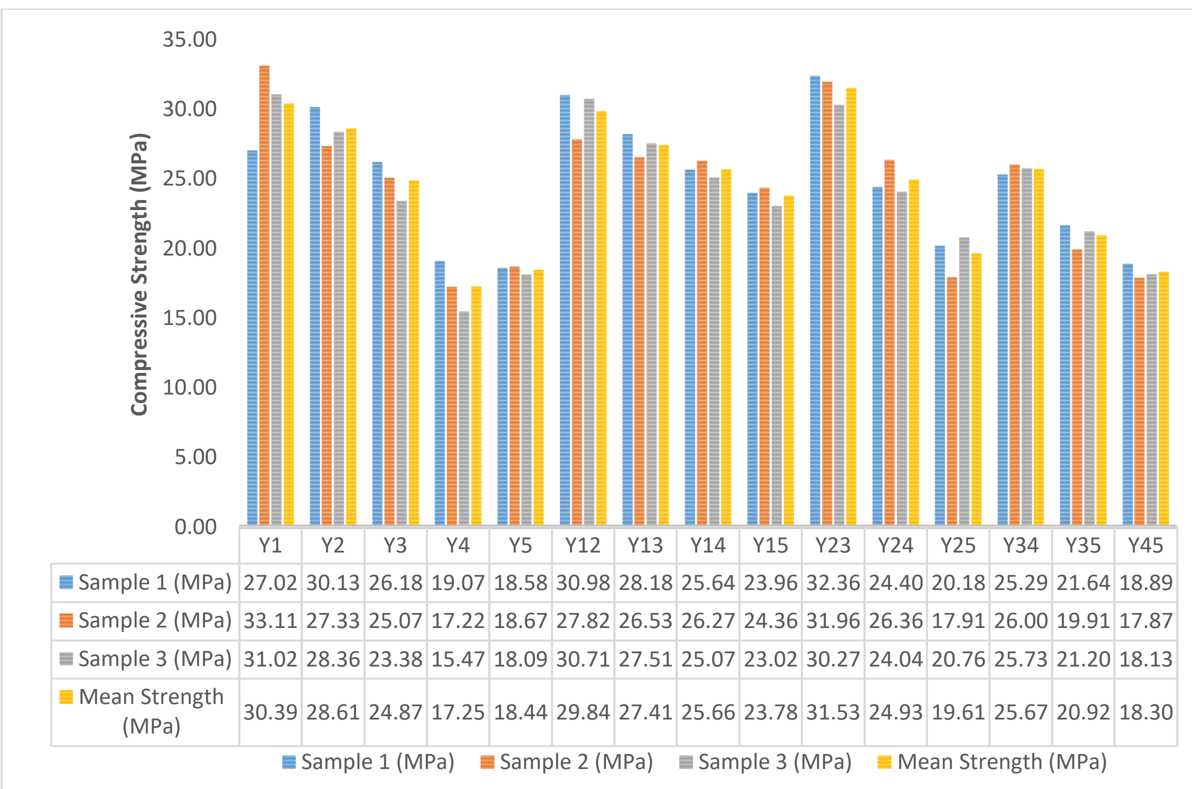
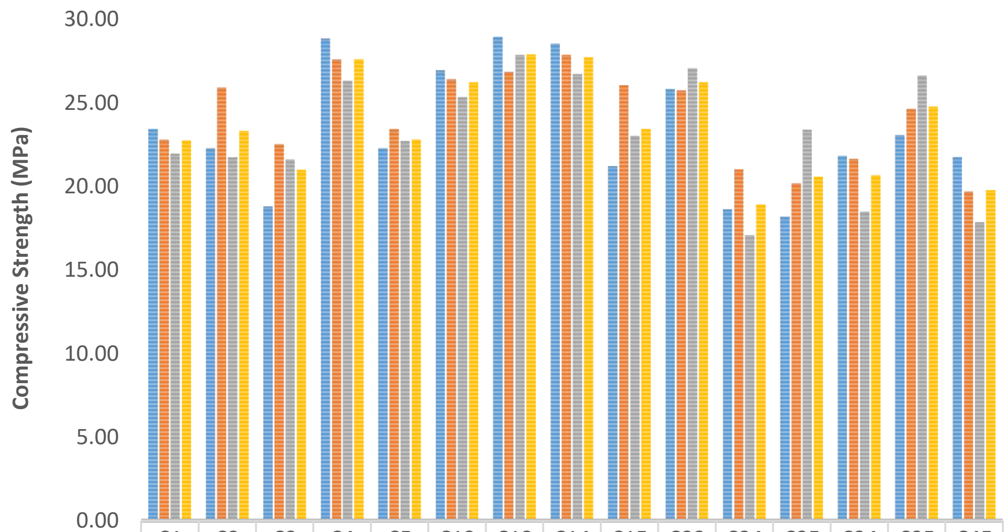


Fig. 2. The Compressive Strength (Laboratory Response, Yi).

The laboratory responses for the control points which were also cured for 28 days for the fifteen design points are presented in fig. 3 below; from the results, points corresponding to C_{14} and C_4 produced the maximum compressive strength value at 27.7 MPa and 27.5 MPa while point corresponding to C_{24} and C_{45} produced the minimum compressive strength values of 18.9 MPa and 19.7 MPa respectively.



	C1	C2	C3	C4	C5	C12	C13	C14	C15	C23	C24	C25	C34	C35	C45
Sample 1 (MPa)	23.4	22.2	18.8	28.8	22.2	26.9	28.9	28.5	21.2	25.8	18.6	18.1	21.8	23.0	21.7
Sample 2 (MPa)	22.8	25.9	22.5	27.6	23.4	26.4	26.8	27.8	26.0	25.7	21.0	20.1	21.6	24.6	19.6
Sample 3 (MPa)	21.9	21.7	21.6	26.3	22.7	25.3	27.8	26.7	23.0	27.0	17.0	23.3	18.4	26.6	17.8
Mean Strength (MPa)	22.7	23.3	20.9	27.5	22.8	26.2	27.8	27.7	23.4	26.2	18.9	20.5	20.6	24.7	19.7

Sample 1 (MPa) Sample 2 (MPa) Sample 3 (MPa) Mean Strength (MPa)

Fig. 3. The Compressive Strength (Laboratory Response, Yi) for the Control points.

4.4. Regression equation for compressive strength

The model equation is generated firstly by substituting the response values in eqn. 9 which shows the relationship between the obtained response and the model coefficients to obtain the Scheffe's coefficients. After which these coefficients values are substituted into Eqn. (7) to give us the model equation shown in eqn. 15 below;

$$\hat{Y} = 30.39X_1 + 28.61X_2 + 24.87X_3 + 17.25X_4 + 18.44X_5 + 1.36X_1X_2 - 0.89X_1X_3 + 7.37X_1X_4 - 2.55X_1X_5 + 19.14X_2X_3 + 8.02X_2X_4 - 15.64X_2X_5 + 18.45X_3X_4 - 2.96X_3X_5 + 1.80X_4X_5 \quad (15)$$

4.5. Test results and replication variance

Mean responses, Y and the variances of replicates Si^2 were obtained from Eqns. (16 - 19) below

$$Y = \frac{\sum_{i=1}^{n} Y_i}{n}; \quad (16)$$

$$Si^2 = \left[\frac{1}{n-1}\right]\left[\sum Y_i^2 - \left[\frac{1}{n(\sum Y_i)^2}\right]\right] (17)$$

Where $1 \leq i \leq n$. The eqn. is expanded as follows;

$$Si^2 = \left\lfloor \frac{1}{n-1} \right\rfloor \left[\sum_{i=1}^{n} [Y_I - Y]^2 \right] \tag{18}$$

Where Y_i represents the laboratory responses; Y represents the average laboratory responses for each experimental point; n is the counts or observations at every point; (n – 1) is the degrees of freedom; Si^2 represents the variance at each design point.

For all the design points, number of degrees of freedom,

$V_e = (\sum n) - 2 = 30 - 2 = 28$

4.5. Replication variance for compressive strength

Table 6 presents the laboratory results and computation of the replication variance at each design point.

$$sy^2 = \left(\frac{1}{V_e}\right) \sum_{i=1}^{N} Si^2 \tag{19}$$

$Sy^2 = 69.61/28 = 2.485901$

Where Si^2 is the variance at each point

$Sy = 1.576674$

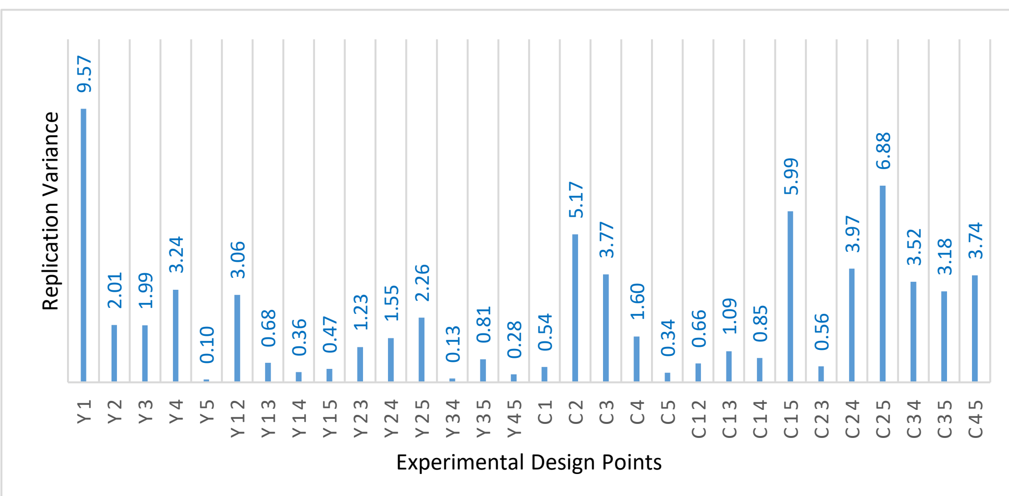


Fig. 4. The Replication Variance of the Experimental Test Result.

4.6. Scheffe's model test for adequacy and validation

The control points of the experiment will be used to test suitability or validity of the model. This adequacy test of the model is carried out using statistical tool for determining differences among means using hypothesis. Analysis of variance ANOVA and The student's t-test method was the statistical tool used. The values generated from the model for the control points which were gotten by substituting the corresponding pseudo-components values X_1, X_2, X_3, X_4 and X_5

Scheffe's model equation expressed in Eqn. (15). Statistical analysis were carried out to test the statistical significance between the experimental and model results shown in Fig. 5 below. The test for adequacy of the model was done using ANOVA and student's t-test at 95% confidence level on the data sets.

4.6.1. Null hypothesis

There is no significant difference between the experimental results and the values predicted by the generated Scheffe's model.

4.6.2. Alternative hypothesis

There is a significant difference between the experimental results and the values predicted by the generated Scheffe's model.

The control experimental values and the obtained control model results are plotted in the graph presented in Fig. 5; these two data sets are compared statistically the test statistical significance between them using student's t-test and analysis of variance (ANOVA).

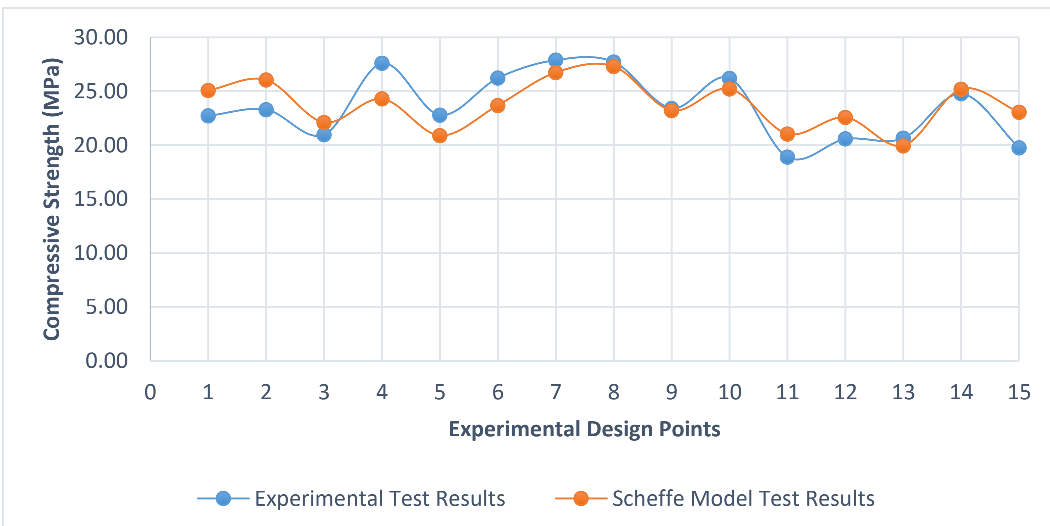


Fig. 5. The Experimental and Model Results for the Control Points.

4.6.4. Student's t-test compressive strength property

A two-tail student's test is used to test the two means and from the results, if calculated t Stat is greater t Critical two-tail, we accept the alternate hypothesis. From the result, t stat value is -0.36331 calculated using the formula below;

$$t_{stat} = \frac{\sum(\mathbf{lab-model})}{\sqrt{\frac{(15*\sum((\mathbf{lab-model})^2)-(\sum(\mathbf{lab-model}))^\wedge 2}{(15-1)}}} \tag{20}$$

$$\frac{(-2.84)}{\sqrt{\frac{(15*57.69)-(-2.84^2)}{(15-1)}}} = \mathbf{-0.36331}$$

Critical value = 0.05 and 0.025 for two tail (t-distribution table).

$t_{critical}$ value obtained from t-distribution table **2.145;** from the obtained results, we observe that t_{stat} is less than $t_{critical}$ value. Therefore, we reject the alternate hypothesis that there is no significant difference between the lab and model results.

4.6.5. Analysis of variance for compressive strength property

For the ANOVA test, if F-value > F crit, we accept the alternate hypothesis. From the calculated result, F-value and F crit of 0.038285 and 4.195972 respectively. This indicates that F crit is greater than F-value; therefore, we accept null hypothesis. This indicates a good relationship among the measured and predicted values; hence, the Scheffe's model generated is adequate for use in compressive strength estimation when the mix ratio is known and vice-versa.

4.6.6. Durability test

The cube samples (Y_2) was soaked for 14 days and then crushed; the compressive gotten were compared with the result gotten under absolute condition using eqns. 13 and 14. 1.1% and 1.2% of Reduction in strength and Weight loss respectively as shown in Table 5 below.

Table 5
The Reduction in Strength and Weight Loss for the Response Parameters.

Compressive Strength				**Durability**	
f'_{abs} (N/mm^2)	f'_s (N/mm^2)	W_{abs} (kg)	W_s (kg)	Reduction in strength	Weight loss
28.21	27.9	8.1	8	1.1%	1.2%

4.7. Discussion of results

Scheffe's simplex lattice model was utilized to evaluate the mechanical property of the fiber reinforced concrete with respect to its compressive strength. The optimum compressive strength of 31.53Nmm^2 corresponding to mix ratio of 0.525**:**1.0**:**1.45**:**1.75**:**0.6 for water, Limestone Portland cement LPC, fine aggregate, coarse aggregate and palm-nut fiber respectively was obtained within the factor space. The lowest compressive strength obtained was 17.25Nmm^2 corresponding to mix ratio of 0.6**:**1.0**:**1.8**:**2.5**:**1.2.

The addition of fiber as a fifth component significantly enhanced the compressive strength of the concrete sample by improving its ductility property. The minimum and maximum value of compressive strength was achieved by addition of 14.667% and 11.2676% percent of fiber respectively. The bond of the natural fibers in composites is very satisfactory.

From the durability test results, the fiber concrete's compressive strength reduction is 1.1% with a weight loss of 1.2%. This result indicates a better durability performance of the concrete, this property reduces the permeability and shrinkage in concrete.

By using the developed mathematical model, the compressive strength of all the design points in the factor space can be derived. This is an application of Scheffe's theory to design a five component concrete using second order polynomial. Fibers are incorporated in concrete mix to help improve its behavior in terms of crack control due to plastic and drying shrinkage; they also help to improve the permeability property of the concrete and thus reduce bleeding of water.

5. Concluding notes

- Scheffe's second order regression polynomial model was developed for optimization of the palm-nut fibre concrete's compressive strength; the generated model was able to estimate its compressive strength when the mix proportions are provided and vice versa.
- The test of adequacy of the generated model was done using analysis of variance (ANOVA) and student's t-test; the statistical results obtained indicate that there is a good relationship between the control laboratory values and computed model results.
- Since the fibres were added by weight basis, this reduces the impact of aspect ratio with respect to mixture density. It was observed generally that the addition of more fibres by weight, leads to reduction in the workability of the concrete mix.
- The maximum compressive strength obtained for the designed factor space is 31.53Nmm2 with a mix proportion corresponding to mix ratio 0.525:1.0:1.45:1.75:0.6 for water, cement, fine and coarse aggregate and palm nut fibre respectively. The lowest compressive strength was found to be 17.25Nmm^2 corresponding to mix ratio of 0.6:1.0:1.8:2.5:1.2.
- There is a saving in cost of when some percentage of palm nut fibre are used and very important in the area of waste management and Concrete made with palm nut fibre are lighter than the normal concrete.

NOTATIONS

q =number of components
k = degree of dimensional space
X_i = proportion of ith components of mixtures
m = order of the Scheffe's polynomial
X_1 = proportion of water cement ratio
X_2 = proportion of ordinary Portland cement
X_3 = proportion of fine aggregate
X_4 = proportion of coarse aggregate
X_5 = proportion of palm nut fiber
n = order of polynomial regression
Z = actual components
X = pseudo components
Y_1, Y_2, Y_3, Y_4, Y_5, Y_{12}, Y_{13}, Y_{14}, Y_{15}, Y_{23}, Y_{24}, Y_{25}, Y_{34}, Y_{35}, Y_{45} = responses from treatment mixture proportions
C_1, C_2, C_3, C_4, C_5, C_{12}, C_{13}, C_{14}, C_{15}, C_{23}, C_{24}, C_{25}, C_{34}, C_{35}, C_{45} = responses from control mixture proportions
β_1, β_2, β_3, β_4, β_5, β_{12}, β_{13}, β_{14}, β_{15}, β_{23}, β_{24}, β_{25}, β_{34}, β_{35}, β_{45}, = model coefficients
Y = optimized compressive strength of palm nut fiber concrete

Conflict of Interests

There are no recorded conflicts of interests in this research work. We also affirm that the content of this work is original and has followed the journal template. Compliance with Ethical Standards was strictly observed.

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